The purpose of these notes is to provide a concise overview of the material covered in MATH 1001 at Georgia State University. Note that the chapters are presented in order that they are presented in the course due to exam scheduling.

Chapter 8

8.1

In this chapter we are interested in percentages, namely being able to convert between the percent form and decimal form.
To express a fraction as a percent, follow these three steps.

1. Divide numerator by denominator, i.e. get a decimal if you have a fraction to start with.
2. Multiply by 100, i.e. move the the decimal two spots to the right.
3. Add the percent sign $\%$.

Example 1. *Express* $\frac{5}{8}$, $\frac{1}{4}$, *and* $.43$ *as percentages.*

$$\frac{5}{8} = .625 \qquad \Rightarrow \qquad .625 \times 100 = 62.5 \qquad \Rightarrow \qquad 62.5\%$$

$$\frac{1}{4} = .25 \qquad \Rightarrow \qquad .25 \times 100 = 25 \qquad \Rightarrow \qquad 25\%$$

In the last case, if you start with a decimal, skip to step two.

$$.43 \times 100 = 43 \qquad \Rightarrow \qquad 43\%$$

More important, for the topics ahead is that we want to be able to convert from the percentage form to the decimal form. Which can be done as follow

1. Move the decimal two spots the left, similar to dividing by 100.
2. Remove the $\%$ sign

Example 2. *Express* 19%, 180%, *and* $\frac{1}{4}\%$ *as a decimal.*

$$19\% \qquad \Rightarrow \qquad \frac{19}{100} = .19$$

$$180\% \qquad \Rightarrow \qquad \frac{180}{100} = 1.8$$

If you are given a percentage that is a fraction, first divide the numerator by the denominator, and then follow the above steps. For example,

$$\frac{1}{4}\% = .25\% \qquad \Rightarrow \qquad \frac{.25}{100} = .0025$$

Recall from chapter one that we had a statement regarding percentages, namely that

$$A = P \cdot B \qquad \text{A is P percent of B.}$$

The first application we are interested in is for calculating sales tax on an item. This can be done in the following way, the tax amount on a item is given by

$$\text{tax amount} = \text{tax rate} \times \text{item cost.}$$

The thing to note is that the tax rate must be in the decimal form!

Example 3. *The local sales tax in Titus is* 7.5% *and Toph is looking to purchase a bicycle for* $\$894$. *How much sales tax will Toph have to pay and how much will she pay in total?*

Note that the tax rate of 7.5% *as a decimal is* $.075$. *Therefore the tax amount is given by*

$$\textit{Tax Amount} = \textit{tax rate} \times \textit{item cost} = (.075) \times (894) = \$67.05$$

Meaning that Toph will have to pay $\$67.05$ *in taxes, and to find her total we just need to add the tax amount and the cost of the item.*

$$\textit{Total} = \textit{tax amount} + \textit{item cost} = \$67.05 + \$894 = \$961.05$$

Example 4. *The local sales tax in Morganville is* 6% *and Sonya is looking to purchase a computer for* $\$1260$. *How much sales tax will Sonya have to pay and how much will she pay in total?*

Note that the tax rate of 6% *as a decimal is* $.06$. *Therefore the tax amount is given by*

$$\textit{Tax Amount} = \textit{tax rate} \times \textit{item cost} = (.06) \times (1260) = \$75.60$$

Meaning that Sonya will have to pay $\$75.60$ *in taxes, and to find her total we just need to add the tax amount and the cost of the item.*

$$\textit{Total} = \textit{tax amount} + \textit{item cost} = \$75.60 + \$1260 = \$1335.60$$

Another application similar to finding the tax amount is finding a discount amount. This is done by

$$\text{Discount Amount} = \text{discount rate} \times \text{original price}$$

where the discount rate is in the decimal form.

Example 5. *The local computer store, Toph Buy, is selling a computer originally priced at* $\$1460$ *for* 15% *off the original price. How much is the discount amount for the computer, and what is the sale price?*

Note that the discount rate of 15% *as a decimal is* $.15$. *Therefore the discount amount is given by*

$$\textit{Discount Amount} = \textit{discount rate} \times \textit{item cost} = (.15) \times (1460) = \$219$$

Meaning that Toph will deduct $\$219$ *from the computer's original price, and to find sale we just need to subtract the discount amount and the cost of the item.*

$$\textit{Total} = \textit{item cost - discount amount} = \$1460 - \$219 = \$1241$$

In life, percents are used to compare changes. If a quantity changes, we may be interested in knowing the percent increase or decrease. This percent, as a decimal, can be found by

$$\frac{\text{amount change}}{\text{original amount}}.$$

Note that here we will take the amount change to always be positive.

Example 6. *In 2000, the world population was found estimated to be around 6 billion people and scientist created three projections as to what the population will be in 2150 that are given as follow:*

1. *High Projection: 30 billion people*
2. *Medium Projection: 13 billion people*
3. *Low Projection: 4 billion people*

Find the percent increase or decrease for each projection.

1. *The high projection goes from 6 billion in 2000 to 30 billion in 2150, so it is a percent increase.*

   Percent increase is calculated by
   $$\frac{\textit{amount change}}{\textit{original amount}} = \frac{30-6}{6} = 4$$
   Remember that 4 is a decimal, so as a percent that is 400% *increase.*

2. *The medium projection goes from 6 billion in 2000 to 13 billion in 2150, so it is a percent increase.*

Percent increase is calculated by

$$\frac{\textit{amount change}}{\textit{original amount}} = \frac{13-6}{6} \approx 1.17$$

Remember that 1.17 is a decimal, so as a percent that is 117% *increase.*

3. *The low projection goes from 6 billion in 2000 to 4 billion in 2150, so it is a percent decrease. Note that we want the precent decrease to still be positive.*

   Percent increase is calculated by

$$\frac{\textit{amount change}}{\textit{original amount}} = \frac{6-4}{6} \approx .33$$

   Remember that .33 is a decimal, so as a percent that is 33% *decrease.*

One should note that a percent decrease can never exceed 100%, but a percent increase can be any percent greater than 0. Also, we can find percent increase and decrease for a myriad of objects, including percents themselves! Consider the following two examples.

Example 7. *A jacket at Hiccup-Mart originally sold for* $\$135$ *is on sale for* $\$60.75$. *What is the percent decrease of the jacket?*

$$\frac{\textit{amount change}}{\textit{original amount}} = \frac{\$135 - \$60.75}{\$135} = .55$$

Remember that .55 is a decimal, so as a percent that is 55% *decrease.*

Example 8. *The audience of the PBS classic, Kitty-Cat Street, is* 13% *versus the usual* 4% *audience for the other PBS shows. What is the percent increase for the audience growth?*

$$\frac{\textit{amount change}}{\textit{original amount}} = \frac{13\% - 4\%}{4\%} = 2.25$$

Remember that 2.25 is a decimal, so as a percent that is 225% *decrease.*

8.3

In the remaining sections of chapter 8 with will be looking at different formulas involving interest, future value, and present value.

Definition 9. ***Interest*** *is the amount of money we are paid when lending or investing money, or the amount of money we pay for borrowing money.*

The amount of money that we deposit, lend, or borrowed is called the ***principal****, denoted by* P.

The first type of interest we are interested (ha), is **simple interest**, denoted I. This type of interest only depends on the principal and interest rate, it is defined by

$$I = Prt,$$

where P is the principal, r is the interest rate as a decimal and t is time in years.

Example 10. *Sonya would like to deposit* $\$2000$ *into a savings account at Titus-One. The account the bank offered has a* 6% *interest rate. How much interest will she have after the first year? What about two years?*

Let us identify the parts we have, according to the problem we have $P = \$2000$, $r = .06$ *and* $t = 1$.*Therefore the interest is given by*

$$I = Prt = (2000)(.06)(1) = 120,$$

which means that the interest earned in the first year is $\$120$. *For the interest earned for two years, we just let* $t = 2$ *now. There fore the interest is given by*

$$I = Prt = (2000)(.06)(2) = 240,$$

which means that interest earned over two years is $\$240$. *Notice that the difference between year two and year one is* $\$240 - \$120 = \$120$, *meaning that this account earns* $\$120$ *each year. Therefore the interest after three years is* $\$120 \times 3 = \$360$.

For banks, they have a trick that they use to calculate certain interest rates, namely the rates related to short-term loans using what is called **banker's rule**. Normally we use a year length of 365 days, however for banker's rule they use 360. The reasoning behind this is two-fold, dividing by 360 is easier and it actually ends up with us having a higher interest rate. For example considering the following.

To find the interest rate for short-term loans, we use the formula

$$\frac{\text{number of days taken}}{360}.$$

If we take a short-term loan for 120 days, then the interest rates, under banker's rule and a normal year length, are given by

$$\frac{120}{360} = .347 \qquad \frac{120}{365} = .342$$

Meaning that under banker's rule, we have an interest rate of 34.7% versus a rate using a normal year length of 34.2%.

The second value that is very important in interest problems is **future value**, denoted A. The future value, can be of a loan, is the principal plus the interest earned. This can be calculated using the formula

$$A = P + I = P + Prt = P(1 + rt)$$

where P is still the principal, r is the rate as a decimal, and t is in years.

Example 11. *Sonya takes a loan from Titus-One of* $\$1060$ *at an interest rate of* 6.5% *for three months. How much will Sonya owe after three months, i.e. what is the loan's future value after three months.*

First, identify what information you have. We have a $P = \$1060$, $r = .065$, *and* $t = 3/12 = .25$. *Note that we were given a time value of months, so we changed it to years by dividing by 12. It then follows that the loan's future value is*

$$A = P(1 + rt) = 2000(1 + (.065)(.25)) \approx \$1077.23$$

To find out how much interest Sonya will pay, we can use the formula $A = P + I$ *and solve for* I, $I = A - P$. *Therefore the interest is* $I = 1077.23 - 1060 = \$17.23$.

Example 12. *Toph borrowed* $\$2500$ *from Titus and promised to pay him back* $\$2655$ *in six months. What is the interest rate that Toph is proposing for her loan?*

First, identify what information we have. We have $P = \$2500$, $A = \$2655$ *since this is how much she is paying back, and* $t = 6/12 = .5$ *since the time-frame was six months. We have a couple options on solving for* r, *we can use the future value formula, plug in the information given and solve for* r *or solve for* r *from the beginning.*

$$r = \frac{A - P}{Pt} = \frac{2655 - 2500}{(2500)(.5)} = .124$$

which remember is in its decimal form. Therefore Toph is proposing a 12.4% *interest rate.*

Example 13. *Hiccup is planning to save* $\$2000$ *in an account earning* 4% *interest by placing a principal deposit* P. *If she wants to achieve* $\$2000$ *in two years, how much should she deposit today?*

First, identify what information we have. We have $A = \$2000$ *since she wants to have that amount in two years,* $r = .04$, *and* $t = 2$. *Similar to the other example we have options on solving for* P *here. The easiest is*

$$P = \frac{A}{(1 + rt)} = \frac{2000}{(1 + (.04)(2))} \approx \$1851.85$$

Therefore Hiccup would need to deposit $\$1851.85$ *to meet her goal in two years.*

8.4

In this section we are interested in **compound interest** which is interest computed on original principal and any accumulated interest. To calculate compound interest, **paid once a year**, we utilize the formula

$$A = P(1 + r)^t.$$

Where A is the future value, P is the present value, r is a rate as a decimal, and t is time in years.

Example 14. *Sonya deposits* $\$2000$ *into a savings account at Titus-One at* 6% *interest compounded yearly. Find how much is in Sonya's account after three years, and find the total interest.*
First, identify what information we have. We are given $P = \$2000$*,* $r = .06$*, and* $t = 3$ *where we are compounding yearly. It then follows that the loans future value is*

$$A = P(1 + r)^t = 2000(1 + .06)^3 \approx \$2382.03$$

meaning that Sonya's account after three years has $\$2382.03$*. To find the total interest earned, all we need to do is subtract the present value from the future value, i.e.* $I = A - P$*. Thus,*

$$I = A - P = \$2382.03 - 2000 = \$382.03$$

which means that Sonya earned $\$382.03$ *of interest over three years.*

For compound interest we are able to compound over different time periods besides yearly.

Name	Number of Compounds per 1 year
Yearly	n=1
Semiannually	n=2
Quarterly	n=4
Weekly	n=52
Daily	n=365

To take into account this different compounding periods, we are able to use the same formula as earlier, but with a slight modification. The compound interest formula for a present value P compound n times per year and a rate r (as a decimal) over t years is

$$A = P\left(1 + \frac{r}{n}\right)^{nt}.$$

Example 15. *Toph is depositing* $\$7500$ *into her savings account at Titus-One that has a* 6% *interest rate that is compounded monthly. How much money will her account have after five years and how much of it is interest?*

First, identify what information is given. We are given $P = \$7500$*,* $r = .06$*,* $n = 12$ *for monthly, and* $t = 5$*. Therefore the future value is given by*

$$A = P\left(1 + \frac{r}{n}\right)^{nt} = 7500\left(1 + \frac{.06}{12}\right)^{12\times 5} \approx \$10,116.38$$

meaning that Toph will have $\$10,116.38$ *in her account after five years. To find the total interest earned we utilize the formula* $I = A - P$*. Thus*

$$I = A - P = 10,116.38 - 7500 = \$2616.38$$

is earned in interest over the five years.

For compound interest, there is a special circumstance hat can occur, namely when we **compound continuously**. This means that we are always compounding, what happens here is that as

$$n \to \infty \qquad \left(1 + \frac{r}{n}\right)^n \to e$$

which is the natural base we discussed in past chapters. Therefore the future value A of a present value P **compounded continuously** at rate r (in decimal form) over t years is

$$A = Pe^{rt}.$$

If we consider the previous example, but now compound continuously over five years yields a future value

$$A = Pe^{rt} = 7500e^{.06\times 5} \approx \$10,123.94$$

which means that the total interest earned in this case is

$$I = A - P = 10123.94 - 7500 = \$2,623.94.$$

In this case Toph earns more interest that the compounded monthly account.

The other question that can be asked is related to the present value P. Meaning, if A is to be accumulated in t years with a rate r (as a decimal) compounded n times, the present value P needs to be

$$P = \frac{A}{\left(1 + \frac{r}{n}\right)^{nt}}.$$

Example 16. *How much should Hiccup deposit today at her Titus-One account earning* 6% *compounded monthly, so that she has* $\$20,000$ *after 5 years?*

First, identify what information we have. We have $A = 20,000$*,* $r = .06$*,* $n = 12$*, and* $t = 5$*. There fore the present value, how much she needs to deposit is*

$$P = \frac{A}{\left(1+\frac{r}{n}\right)^{nt}} = \frac{20000}{\left(1+\frac{.06}{12}\right)^{12\times 5}} \approx 14,827.44$$

meaning that to have her goal in five years, Hiccup needs to deposit $\$14,827.44$ *today.*

The other type of question we are interested in relates to **effective rate** sometimes called effective annual yield. This is a simple interest rate that produces the same amount of interest after one year as an account subject to compound interest at a stated rate. The formula for annual yield, Y, under a nominal rate r (as a decimal), this is the rate we compound, compounded n times per year is given by

$$Y = \left(1+\frac{r}{n}\right)^{n} - 1.$$

Example 17. *Titus has* $\$4,000$ *in an account compounding monthly at a* 8% *interest rate, what is the effective rate of the account.*

For this type of problem, all we need is the nominal rate $r = .08$ *and* $n = 12$*. Therefore the effective rate is*

$$Y = \left(1+\frac{r}{n}\right)^{n} - 1 = \left(1+\frac{.08}{12}\right)^{1} 2 - 1 \approx .08299$$

which means the effective rate is about 8.3%*.*

8.5

In this section we are going to look at the topic of annuity. An **annuity** is a sequence of equal payments that are made at equal time periods. Even though it seems like a highly abstract notion, we deal with annuities quite often. Typically we see annuities as IRAs (Individual Retirement Agencies), but we can also see them as car payments, house notes, etc.

The value of an annuity is the sum of all deposits and the interest paid. More precisely, if a deposit P is made at the end of each year for an annuity paying an annual interest rate r, compounded yearly, then the value A of the annuity after t years is given by

$$A = \frac{P[(1+r)^t - 1]}{r}.$$

Example 18. *Sonya is looking to retire, and is thinking about making* $\$1000$ *deposits into an IRA at the end of each year for thirty years. If her IRA has an interest rate of* 10% *compounded yearly, how much will she have in the annuity after thirty years and how much is interest?*

First, find all the information that is given. Sonya is depositing $P = 1000$*, the rate is* $r = .10$ *and the time length is* $t = 30$*. Therefore the future value of the annuity is given by*

$$A = \frac{1000[(1+.1)^{30} - 1]}{.1} \approx 164,494$$

which means after thirty years the annuity will be worth $\$164,494$*. To find how much is interest, all we need to do is subtract the total deposit amount from* A*. Meaning*

$$I = A - Pt = 164,494 - (1000\times 30) = \$134,494$$

is made from interest.

Similar to compound interest, we can make deposits over different periods, which gives us a different formula. Thus, if a P deposit is made at the end of each compound for an annuity that pays an annual rate r (as a decimal) compounded n times per year, the value A, of the annuity after t years is

$$A = \frac{P\left[\left(1+\frac{r}{n}\right)^{nt} - 1\right]}{\left(\frac{r}{n}\right)}.$$

Example 19. *Toph at the age of 25 is looking to start an IRA and wants to deposit $\$200$ at the end of each month. If the IRA pays 7.5% compounded monthly, how much will she have in the IRA at age 65 and how much is from interest?*

First, find all the information that is given. Toph is depositing $P = 200$, the rate is $r = .75$, $n = 12$, and the time length is $t = 65 - 25 = 40$. Therefore the future value of the annuity is given by

$$A = \frac{200\left[\left(1 + \frac{.75}{12}\right)^{12\times 40} - 1\right]}{\left(\frac{.75}{12}\right)} \approx 604,765$$

which means after forty years the annuity will be worth $\$604,765$. To find how much is interest, all we need to do is subtract the total deposit amount from A. Meaning

$$I = A - Pnt = 604,765 - (200 \times 12 \times 40) = \$508,765$$

is made from interest.

Example 20. *Hiccup at the age of 30 is looking to start an IRA and wants to deposit $\$100$ at the end of each month. If the IRA pays 9.5% compounded monthly, how much will she have in the IRA at age 65 and how much is from interest?*

First, find all the information that is given. Hiccup is depositing $P = 100$, the rate is $r = .95$, $n = 12$, and the time length is $t = 65 - 30 = 35$. Therefore the future value of the annuity is given by

$$A = \frac{200\left[\left(1 + \frac{.95}{12}\right)^{12\times 35} - 1\right]}{\left(\frac{.95}{12}\right)} \approx 333,946$$

which means after thirty-five years the annuity will be worth $\$333,946$. To find how much is interest, all we need to do is subtract the total deposit amount from A. Meaning

$$I = A - Pnt = 333,946 - (100 \times 12 \times 35) = \$291,946$$

is made from interest.

The other value we are interested in is the value P which is called a deposit for annuities. The deposit P that must be made at the end of each compounding period into an annuity paying an interest rate r (as a decimal) compounded n times per year, then to achieve the value A after t years is given by

$$P = \frac{A\left(\frac{r}{n}\right)}{\left[\left(1 + \frac{r}{n}\right)^{nt} - 1\right]}.$$

Example 21. *Suppose you want to save $\$20,000$ over five years to use as a down payment for a home. You anticipate making regular, end of the month, deposits in an annuity that pays 6% compounded monthly. How much should you deposit to meet your goal and how much is from deposits?*

First, find all of the information that is given. You have $A = 20,000$, $r = .06$, $n = 12$, and $t = 5$. Thus the required deposit is given by

$$P = \frac{20000\left(\frac{.06}{12}\right)}{\left[\left(1 + \frac{.06}{12}\right)^{12\times 5} - 1\right]} \approx 287$$

meaning that we need to deposit $\$287$ to hit our goal. To find how much is from deposits, it follows

$$\textit{Total Deposits } = Pnt = 287 \times 12 \times 5 = 17,220$$

meaning that $17,720$ is from deposits and $I = A - Pnt = \$2,780$ from interest.

Example 22. *Suppose you want to save $\$100,000$ over eighteen years to pay for college. You anticipate making regular, end of the month, deposits in an annuity that pays 9% compounded monthly. How much should you deposit to meet your goal and how much is from deposits?*

First, find all of the information that is given. You have $A = 100,000$, $r = .09$, $n = 12$, and $t = 18$. Thus the required deposit is given by

$$P = \frac{100000\left(\frac{.09}{12}\right)}{\left[\left(1 + \frac{.09}{12}\right)^{12\times 18} - 1\right]} \approx 187$$

meaning that we need to deposit $\$187$ to hit our goal. To find how much is from deposits, it follows

$$\textit{Total Deposits } = Pnt = 187 \times 12 \times 18 = 40,392$$

meaning that $40,392$ is from deposits and $I = A - Pnt = \$59,608$ from interest.

8.6

A loan you pay off weekly or monthly, or any other time period is called an installment plan. For cars, you make regular monthly payments called a fixed installment loan. To find the regular monthly payments, PMT, required to pay off a loan of P dollars paid n times per year over t years at an annual rate r (as a decimal) is given by

$$PMT = \frac{P\left(\frac{r}{n}\right)}{\left[1-\left(1+\frac{r}{n}\right)^{-nt}\right]}.$$

Example 23. *Sonya decided to borrow* $\$20,000$ *for a new Purr-cedes Benz. She has two options on loans, requiring regular monthly payments. Loan A is a rate of* 7% *for 3 years and loan B is 5 years at* 9%. *Find the month payments and total interest for both loans.*

Find the information for each loan. For Loan A, $P = 20,000$, $r = .07$, $n = 12$ *and* $t = 3$, *therefore the regular monthly payments is*

$$PMT = \frac{20000\left(\frac{.07}{12}\right)}{\left[1-\left(1+\frac{.07}{12}\right)^{-12\times 3}\right]} \approx 618$$

meaning that you have to pay $\$618$ *per month. The interest for the loan is*

$$I = PMTnt - P = 2248 - 20000 = \$2248$$

in interest. The other loan, the only change on the variable is $r = .09$ *and* $t = 5$, *therefore the regular monthly payments is*

$$PMT = \frac{20000\left(\frac{.09}{12}\right)}{\left[1-\left(1+\frac{.09}{12}\right)^{-12\times 5}\right]} \approx 415$$

meaning you need to pay $\$415$ *per month. The interest for the loan is*

$$I = PMTnt - P = 24900 - 20000 = \$4900$$

in interest.

Example 24. *Hiccup decided to borrow* $\$15,000$ *for a new Furiat. She has two options on loans, requiring regular monthly payments. Loan A is a rate of* 8% *for 4 years and loan B is 6 years at* 10%. *Find the month payments and total interest for both loans.*

Find the information for each loan. For Loan A, $P = 15,000$, $r = .08$, $n = 12$ *and* $t = 4$, *therefore the regular monthly payments is*

$$PMT = \frac{15000\left(\frac{.08}{12}\right)}{\left[1-\left(1+\frac{.08}{12}\right)^{-12\times 4}\right]} \approx 366$$

meaning that you have to pay $\$366$ *per month. The interest for the loan is*

$$I = PMTnt - P = 17568 - 15000 = \$2568$$

in interest. The other loan, the only change on the variable is $r = .10$ *and* $t = 6$, *therefore the regular monthly payments is*

$$PMT = \frac{15000\left(\frac{.1}{12}\right)}{\left[1-\left(1+\frac{.1}{12}\right)^{-12\times 6}\right]} \approx 278$$

meaning you need to pay $\$277$ *per month. The interest for the loan is*

$$I = PMTnt - P = 20,016 - 15000 = \$5016$$

in interest.

Example 25. *Suppose you are interested in a new car. Car A is a new car costing* $\$25,000$ *and can be financed with a four year loan* 7.9%. *The other car, B, is a used car costing* $\$14,000$ *and can be financed with a four year loan at* 8.45%. *What is the difference in regular monthly payments?*

Find the information for each car. For car A, $P = 25,000$, $r = .079$, $n = 12$ *and* $t = 4$, *therefore the regular monthly payments is*

$$PMT = \frac{25000\left(\frac{.079}{12}\right)}{\left[1-\left(1+\frac{.079}{12}\right)^{-12\times 4}\right]} \approx 609$$

meaning that you have to pay $\$609$ per month. The interest for the loan is

$$I = PMTnt - P = 29232 - 25000 = \$4,232$$

in interest. The other car, the only change on the variable is $P = 14,000$, $r = .0845$ and $t = 4$, therefore the regular monthly payments is

$$PMT = \frac{14000\left(\frac{.0845}{12}\right)}{\left[1 - \left(1 + \frac{.0845}{12}\right)^{-12\times 4}\right]} \approx 345$$

meaning you need to pay $\$345$ per month. The interest for the loan is

$$I = PMTnt - P = 16560 - 14000 = \$2,560$$

in interest.

The difference in monthly payments is $609 - 345 = \$264$.

Example 26. *Suppose you drive 24,000 miles per year and the average gas price of gas is $\$4$ per gallon. What is the annual fuel expense of a hybrid with 50 mpg versus a SUV with 12 mpg. If you deposit your monthly savings at the end of each month in an annuity paying 7.3% interest compounded monthly, how much will you save after six years.*

The annual fuel cost can be found by

$$\textit{Fuel Cost} = \frac{\textit{annual miles driven}}{\textit{mpg}} \times \textit{(price per gallon)}$$

meaning that the fuel cost for the hybrid is given by $(24,000/50) \times 4 = \$1920$ versus the SUV's fuel cost is given by $(24,000/12) \times 4 = \$8,000$. The amount you save per month is given by

$$\frac{8000 - 1920}{12} = \frac{6080}{12} \approx \$507.$$

Therefore the future value of the annuity is given by

$$A = \frac{507\left[\left(1 + \frac{.073}{12}\right)^{12\times 6} - 1\right]}{\left(\frac{.073}{12}\right)} \approx \$45,634.$$

Chapter 8 Formulas

Tax Amount	Tax amount = tax rate $\times$ item cost
Discount Amount	Discount Amount = discount rate $\times$ item cost
Percent Increase/Decrease	Increase/Decrease (decimal) $= \dfrac{\text{amount increase/decrease}}{\text{original amount}}$
Simple Interest, I	$I = Prt$
Short-Term Loan Rate	r (decimal) $=$ (days taken)$/360$
Future Value, Simple, A	$A = P + I = P(1+rt)$
Principal, Simple, P	$P = A/(1+rt)$
Rate, Simple, r	r (decimal) $= (A-P)/(Pt)$
Compound Interest, Yearly,	$A = P(1+r)^t$
Compound Interest, General n	$A = P\left(1+\frac{r}{n}\right)^{nt}$
Compound Interest, Continuous	$A = Pe^{rt}$
Present Value, Compound, P	$P = \dfrac{A}{\left(1+\frac{r}{n}\right)^{nt}}$
Annual Yield, Y	Y (decimal) $= \left(1+\frac{r}{n}\right)^n - 1$
Annuity Future Value Yearly	$A = \dfrac{P[(1+r)^t - 1]}{r}$
Annuity Future Value, General n	$A = \dfrac{P\left[\left(1+\frac{r}{n}\right)^{nt} - 1\right]}{\left(\frac{r}{n}\right)}$
Interest From an Annuity, I	$I = A - Pnt$
Regular Monthly Payments, PMT	$PMT = \dfrac{P\left(\frac{r}{n}\right)}{\left[1-\left(1+\frac{r}{n}\right)^{-nt}\right]}$