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MATH 1001
Quantitative Reasoning

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The purpose of these notes is to provide a concise overview of the material covered in MATH 1001 at Georgia State University. Note that the chapters are presented in order that they are presented in the course due to exam scheduling.

Chapter 9

In this chapter we will look at units of measurement, volume, area, and capacity and ways to convert between them.

9.1

Definition 1. *To **measure** an object is to assign a number to its size. The number representing the measure from end to end is called **length**. The result from measuring length is called **linear measurement** and is stated in linear units.*

There are two types of units, the first we will look at is the Imperial units. The Imperial units of linear measurement and their conversions are given as follows:

12 in (inches) = 1 ft (foot)
3 ft = 1 yd (yard)
36 in = 1 yd
5280 ft = 1 mi (mile)

The process of converting between units is called dimensional analysis which utilizes unit fractions. A **unit fraction** is a fraction with the property that the numerator and denominator have different values and the value is 1. For example the following are unit fractions:

$$\frac{1 \text{ ft}}{12 \text{ in}}; \qquad \frac{3 \text{ ft}}{1 \text{ yd}}; \qquad \frac{5280 \text{ ft}}{1 \text{ mi}}.$$

Note that the fractions represent the length value of 1. To convert to a different unit, multiply by a unit fraction(s). The given unit should be in the denominator so it cancels and the unit to be introduced should be in the numerator.

Example 2. *Convert 5 ft to inches.*

First thing is, do we have a conversion from feet to inches? Yes we do!

$$\frac{5\,ft}{1} \cdot \frac{12\,in}{1\,ft} = 5(12)\,in = 60\,in.$$

Here the unit of feet cancels and we are left with the unit of inches which is what we want.

Example 3. *Convert 158,400 in to miles.*

First thing is, do we have a conversion from inches to miles? We do not, but we can convert to feet first.

$$\frac{158,400\,in}{1} \cdot \frac{1\,ft}{12\,in} \cdot \frac{1\,mi}{5280\,ft} = \frac{154,800}{12 \cdot 5280}\,mi = 2.5\,mi.$$

The other system, the metric system, utilizes a series of prefixes to express the size of what we are doing (length, capacity, etc). These prefixes are standard across all of the different ways to measure, they are as follow:

kilo	k	1000 bu
hecto	h	100 bu
deka	da	10 bu
deci	d	1/10 bu
centi	c	1/100 bu
milli	m	1/1000 bu

For linear measurement the base unit of the metric system is the meter, therefore the full chart is as follows:

kilometer	km	1000 m
hectometer	hm	100 m
dekameter	dam	10 m
meter	m	1 m
decimeter	dm	1/10 m
centimeter	cm	1/100 m
millimeter	mm	1/1000 m

The nice thing about the metric system is that unit conversion is really easy since everything is a multiple on 10.

1. If we want to go from a larger unit to a smaller unit (moving down on the chart) we multiply by 10 for each step. This is akin to moving the decimal to the right for each step we take.

   If we want to convert from hm to dm, we see that we have hm $\rightarrow$ dam $\rightarrow$ m $\rightarrow$ m which is 3 steps, so we multiply by 10^3 or move the decimal point three spots to the right.

2. If we want to go from a smaller unit to a larger unit (moving up on the chart) we divide by 10 for each step. This is akin to moving the decimal to the left for each step we take.

   If we want to convert from cm to dam, we see that we have cm $\rightarrow$ dm $\rightarrow$ m $\rightarrow$ dam which is 3 steps, so we divide by 10^3 or move the decimal point three spots to the left.

Example 4. *Convert 504.7 m to km.*
Looking at the chart we see that to get from meters to kilometers we need to move 3 steps up, which means we need to move the decimal place 3 places to the left.

$$504.7\ m \ \rightarrow 50.47\ dam \ \rightarrow 5.047\ hm \ \rightarrow .5047\ km$$

Example 5. *Convert 704 mm to hm.*
Looking at the chart we see that to get from millimeters to hectometers we need to move 5 steps up, which means we need to move the decimal place 5 places to the left.

$$704\ mm \ \rightarrow 70.4\ cm \ \rightarrow 7.04\ dm \ \rightarrow .704\ m \ \rightarrow .0704\ dam \ \rightarrow .00704\ hm$$

Example 6. *Convert 27 m to cm.*
Looking at the chart we see that to get from meters to centimeter we need to move 2 steps down, which means we need to move the decimal place 2 places to the right.

$$27.\ m \ \rightarrow 270.\ dm \ \rightarrow 2700.\ cm$$

There are ways to convert between the Imperial and metric systems, however some of the ways we will utilize are not exact but rather approximation, albeit good ones. These conversions are as follows:

1 in = 2.54 cm	exact
1 ft = 30.48 cm	exact
1 yd ≈ .9m	approximation
1 mi ≈ 1.6 km	approximation

Example 7. *Convert 8 in to cm.*

$$\frac{8\ in}{1} \cdot \frac{2.54\ cm}{1\ in} = 8 \cdot 2.54\ cm = 20.32\ cm.$$

Example 8. *Convert 125 mi to km.*

$$\approx \frac{125\ mi}{1} \cdot \frac{1.6\ km}{1\ mi} = 125 \cdot 1.6\ km = 200\ km.$$

Note that this was an approximation, but we can convert in an exact manner in the following way

$$\frac{125\ mi}{1} \cdot \frac{5280\ ft}{1\ mi} \cdot \frac{30.48\ cm}{1\ ft} \cdot \frac{1\ km}{10^5\ km} = \frac{125 \cdot 5280 \cdot 30.48}{10^5}\ km = 201.168\ km.$$

Example 9. *Convert 26,800 mm to in.*

$$\frac{26,800 \; mm}{1} \cdot \frac{1 \; cm}{10 \; mm} \cdot \frac{1 \; in}{2.54 \; cm} = \frac{26,800}{10 \cdot 2.54} \; in \approx 1055 \; in.$$

Example 10. *The speed limit on many highways in the United States is 55 miles per hour. How many kilometers per hour?*

Note that we have two units, miles and hours. The hours will appear in the dimensional analysis but will not be used. Since it is miles per hour, the miles is the numerator unit and hours is the denominator unit.

$$\approx \frac{55 \; mi}{1 \; hr} \cdot \frac{1.6 \; km}{1 mi} = \frac{55 \cdot 1.6 \; km}{1 \; hr} = 88 km/h.$$

If you want extra work with these types of conversions, show that 200 km/hr is about 125 mi/h and 60 km/hr is about 37.5 mi/hr.

9.2

Definition 11. *A **square unit** is a square whose sides are 1 unit in length.*

Recall that the area of a square is the length of it side squared, so a 1 in square has area 1 in^2 or 1 sq in. Similarly a square with side length 1 cm has an area of 1 cm^2 or 1 sq cm.

These square units will be used to talk about area and similar to linear measurement we have conversions for Imperial, metric, and between the two systems. The Imperial conversions are given by

1 sq ft = 144 sq in
1 sq yd = 9 sq ft = 1296 sq in
1 acre = 43,560 sq ft = 4,840 sq yd
1 sq mi = 640 acres

Example 12. *The population density of Wyoming is 568,158 with an area of 97,814 sq mi. What is the population density?*

*The **population density** is population divided by area, so the population density of Wyoming is*

$$\frac{568,158 \; people}{97,814 \; sq \; mi} \approx 5.8 \; people/ \; sq \; mi.$$

Similarly, the state of California has a population 39,144,818 with an area of 163,695 sq mi. Therefore the population density is

$$\frac{39,144,818 \; people}{163,695 \; sq \; mi} \approx 239.1 \; people/ \; sq \; mi.$$

The dimensional analysis for area is still done in a similar way to how it was done for linear measurement, however the unit fractions will be different since we need to use the ones for square units.

Example 13. *Convert 75,000 acres to sq mi.*

$$\frac{75,000 \; acres}{1} \cdot \frac{1 \; sq \; mi}{640 \; acres} = \frac{75,000}{640} \approx 117 \; sq \; mi.$$

Similar to linear measurement there are conversions between Imperial and metric square units however all of these are approximation.

1 sq in $\approx$ 6.5 sq cm
1 sq ft $\approx$ .09 sq m
1 sq yd $\approx$ .8 sq m
1 sq mi $\approx$ 2.6 sq km
1 acres $\approx$ .4 ha (hectare)

Example 14. *A property is advertised for* $\$545,000$ *for 18 acres. How many hectares is the property and what is the price per hectare?*

The first question is converting acres to hectares which is done as follows:

$$\frac{18 \; acres}{1} \cdot \frac{.4 \; ha}{1 \; acres} = 18 \cdot (.4) \; ha \approx 7.2 \; ha.$$

The remaining part asks how much is each hectare. If 18 7.2 ha is $\$545,000$*, then 1 ha is*

$$\frac{\$545,000}{7.2 \; ha} \approx \$75,694 \; per \; ha.$$

Definition 15. *The amount of space an object occupies is* ***volume*** *and to measure this space we need cubic units. A cubic unit is a cube where each side is 1 unit in length.*

For this section, we are interested in a type of volume often called **capacity** which refers to how much fluid the object can hold. Some of the common Imperial units are given as

2 pints = 1 quart	1 yd^3 $\approx$ 200 gal
4 quarts = 1 gal	1 ft^3 $\approx$ 7.48 gal
1 gal = 128 fl oz	231 in^3 $\approx$ 1 gal
1 cup = 8 fl oz	

Example 16. *A pool has a volume of 100,000* ft^3*, how many gallons of water can it hold.*

We do have a direct conversion from cubic feet to gallons!

$$\approx \frac{100,000 \, ft^3}{1} \cdot \frac{7.48 \; gal}{1 \, ft^3} = 748,000 \; gal.$$

Note this is an approximation!

For the metric system, the base unit for volume is the liter, denoted L. The new metric chart is as follows:

kiloliter	kL	1000 L
hectoliter	hL	100 L
dekaliter	daL	10 L
liter	L	1 L$\approx$ 1.0567 quarts
deciliter	dL	1/10 L
centiliter	cL	1/100 L
milliliter	mL	1/1000 L

Note, the division and multiplying by 10 trick still works here, so use it! What we are interested in again is the relationship between volume and capacity. For metric, we have some nice conversions, but I will also include some Imperial conversions that are used in baking.

Volume	Capacity
1 cm^3	1 mL
1000 cm^3	1 L
1 m^3	1 kL
1 tsp	$\approx$ 5 mL
1 tbsp	$\approx$ 15 mL
1 fl oz	$\approx$ 30 mL
1 cup	$\approx$ .24 L
1 pt	$\approx$ .47 L
1 qt	$\approx$ .95 L
1 gal	$\approx$ 3.8 L

Example 17. *An aquarium has a volume of 36,000* cm^3*, how many liter of water can it hold.*

We do have a direct conversion from cubic feet to gallons!

$$\frac{36,000 \; cm^3}{1} \cdot \frac{1 \; L}{1000 \; cm^3} = 36 \; L.$$

Note this is exact!

Example 18. *A doctor orders 20 cc of a drug. How many mL? How many fluid ounces?*

The nice things about cc (cubic centimeters) and mL, is that the conversion is one to one. So 20 cc= 20 mL!

Now we do not have a direct conversion from cc to fl oz, but we do from mL to fl oz. So the following occurs

$$\approx \frac{20\ mL}{1} \cdot \frac{1\ fl\ oz}{30\ mL} \approx .67\ fl\ oz.$$

9.3

The final section we will look at is weight and it relationship with volume and temperature conversions.

Definition 19. *The measure of earth's gravitation pull on an object is called* ***weight****, and the measure of the quantity of matter in a structure is called* ***mass****.*

The weight of an object can change depending on location, but the mass cannot!

I will place both Imperial and metric weights together, here the base unit for metric is gram.

kilogram	kg	1000 g
hectogram	hg	100 g
dekagram	dag	10 g
gram	g	1 g
decigram	dg	1/10 g
centigram	cg	1/100 g
milligram	mg	1/1000 g
16 oz	=	1 lbs
2000 lbs	=	1 T (ton)
1 kg	≈	2.2 lbs
.45 kg	≈	1 lbs

Similar to the other sections, the dimensional analysis remains the same, just with different unit fractions. There is a relationship between weight and volume (and therefore capacity):

Volume	Capacity	Weight
1 cm^3	1 mL	1 g
1 dm^3 $= 1000$ cm^3	1 L	1 kg
1 m^3	1 kL	1000 kg = 1 t (tonne)

A tonne is also called a metric tonne, not to be confused with the Imperial unit ton. I will also include some approximations between some common metric units and Imperial units.

1 oz ≈ 28 g
1 lbs ≈ .45 kg
1 T ≈ .9 t

Example 20. *Convert 160 lbs to kg and 300 g to ounces.*

For the first conversion, we get an approximation

$$\approx \frac{160\ lbs}{1} \cdot \frac{.45\ kg}{1\ lbs} = 160 \cdot .45\ kg = 72\ kg.$$

For the second conversion, we are using the relationship between capacity and weight. It goes as follows

$$\approx \frac{300\ g}{1} \cdot \frac{1\ oz}{28\ g} = \frac{300}{28}\ oz \approx 10.6\ oz.$$

Conversion check: Do you get 120 lbs is approximately 54 kg? What about 500 g is approximately 17.9 oz.

Example 21. *Drug doses sometimes depend on the weight of the patient. For example 6 mg of a drug is administered daily for each kilogram of a patient's weight.*

How many 200 mg tablets should be given to a patient that weights 150 lbs?

First thing to notice is that we have a dosage of 6 mg/kg, and the patient's weight in pounds. We need to convert to kilograms first.

$$\approx \frac{150\ lbs}{1} \cdot \frac{.45\ kg}{1\ lbs} \approx 67\ kg.$$

(We rounded down in the approximation on weight). Since the dosage of the drug is 6 mg/ kg and the patient weights 67 kg, then they need

$$\frac{6\ mg}{1\ kg} \cdot \frac{67\ kg}{1} = 402\ mg.$$

We now see that the patient needs 402 mg of the drug based on their weight, so to find how many 200 mg tablets are need, we divide.

$$\frac{402\ mg}{200\ mg} \approx 2\ tablets.$$

The last topic involves temperature conversions, and we will look at Fahrenheit (°F), Celsius (°C), and Kelvin (K) scales. To move between these scales, we have a series of formulas:

$$\begin{aligned} \text{°F to °C:} &\qquad C = \frac{5}{9}(F - 32) \\ \text{°C to °F:} &\qquad F = \frac{9}{5}C + 32 \\ \text{°C to K:} &\qquad K = C + 273.15 \\ \text{K to °C:} &\qquad C = K - 273.15 \end{aligned}$$

For the Kelvins scale, one must convert to Celsius before converting to Kelvins. The special part about the Kelvins scale is that there are no negative Kelvins, 0K is called absolute zero. At absolute zero, the temperature is so cold that the particles have minimum movement and the gas particles have no thermal energy.

Example 22. *Convert* $50°$ *C to Fahrenheit.*

$$F = \frac{9}{5}50 + 32 = 9 \cdot 10 + 32 = 122°F$$

Example 23. *Convert* $52°$ *F to Kelvins.*

First we need to convert to Celsius using our formula.

$$C = \frac{5}{9}(52 - 32) = \frac{5}{9}20 \approx 11.1°C$$

Then we use the conversion to Kelvins

$$K = 11.1 + 273.15 = 284.25\ K.$$