

Final Exam

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Topics to Study:

**Problem 1** Solve  $\sin\left(\arcsin\frac{1}{\sqrt{2}} - \arccos\frac{\sqrt{3}}{2}\right)$  use exact values.

**Solution** Find  $\cos\alpha$  and  $\sin\beta$  (3 pts)

$$\cos\alpha = \frac{1}{\sqrt{2}} \quad \sin\beta = \frac{1}{2}$$

Correct formula and simplification (2 pts)

$$\sin(\alpha - \beta) = \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right) = \frac{\sqrt{3} - 1}{2\sqrt{2}}$$

**Problem 2** Find all all solutions to  $\cos\left(5x + \frac{3\pi}{4}\right) = 1$

**Solution:** Use arc-cosine (3 pts)

$$\alpha = \pm \arccos(1) + 2\pi k = \pm \frac{\pi}{2} + 2\pi k$$

Solve for  $x$  (2 pts)

$$x = -\frac{3\pi}{20} \pm \frac{\pi}{10} + \frac{2\pi k}{5}$$

**Problem 3** Find all solutions to  $6 \tan^2 x - 20 \tan x + 6 = 0$

**Solution:** Find roots (3 pts)

$$6t^2 - 20t + 6 = 0 \quad \Rightarrow \quad 2(t - 3)(3t - 1) = 0$$

Solutions (2 pts),  $t = 3$  no solution

$$t_k = \arctan(1/3) + \pi k$$

**Problem 4** Calculate and find the modulus of  $(3 - \sqrt{2}i)^7$

**Solution:** Find polar form (2 pts)

$$\|\omega\| = \sqrt{13} \quad \theta = -\arccos 3/\sqrt{13} \cong -.588 \quad \sqrt{13} \operatorname{cis}(-.588)$$

Calculate (2 pts)

$$(\sqrt{13} \operatorname{cis}(-.588))^7 = (\sqrt{13})^7 \operatorname{cis}(7(-.588)) = -4407 - 255.972i$$

Calculate modulus (1 pt) 4414.43.

**Problem 5** Solve the triangle  $\triangle ABC$  based on  $a = 6$ ,  $b = 4$  and  $A = 37^\circ$ .

**Solution:** SSA Triangle, Law of sines (1 pt)

Find both  $B_1$  and  $B_2$  (2 pts)

$$B_1 = \arcsin \frac{4 \sin 37}{6} = 23.657^\circ \quad B_2 = 180 - B_1 = 156.343^\circ$$

Only  $B_1$  works (1 pt)

Find  $c$  and  $C$  (2 pts)

$$C = 180 - 23.657 - 37 = 119.343^\circ \quad c = \frac{6 \sin 119.343}{\sin 37} \cong 8.69$$

**Problem 6** Find sine, cosine, and tangent of  $\theta/2$  given  $\tan \theta = \frac{24}{7}$  and  $\pi < \theta < 2\pi$ .

**Solution:** Find cosine (2 pts)

$$r = \sqrt{24^2 + 7^2} = 25 \quad \cos \theta = -7/25$$

Find appropriate answers (3 pts)

$$\sin \theta/2 = \sqrt{\frac{1 + (7/25)}{2}} = 4/5 \quad \cos \theta/2 = -\sqrt{\frac{1 - (7/25)}{2}} = -3/5 \quad \tan \theta/2 = -4/3$$

**Problem 7** Find all solutions using exact values

$$\cos 3x + \cos 5x = 0$$

and

$$\sec 2x - \sec 6x = 0.$$

**Solution:**

$$x_k = \pm \frac{\pi}{8} + \frac{\pi k}{2} \quad x_k = \pm \frac{\pi}{2} + 2\pi k$$

$$x_k = \frac{\pi k}{2}$$

**Problem 8** Find all 5<sup>th</sup> roots of -243i.

**Solution:** Modulus and angle (2 pts)

$$\|\omega\| = 243 \quad \theta = -\pi/2$$

Find roots (3 pts)

$$z_k = \sqrt[5]{243} \operatorname{cis} \left( \frac{-\pi/2 + 2\pi k}{5} \right)$$

**Problem 9** Find all solutions of the given equation

$$x^6 - 6x^3 + 3 = 0$$

**Solution:** Find cubic breakdown (2 pts)

$$x^6 - 6x^3 + 3 = (x^3 - (3 - \sqrt{6}))(x^3 - (3 + \sqrt{6}))$$

Use general roots on both parts (3 pts)

**Problem 10** Given the equation

$$2x^2 + 3x + 5y^2 + 10y = 7,$$

Find the standard form of the ellipses and give all of the relevant data: Horizontal/Vertical Major axis, length of major and minor axis, coordinates of the minor points, vertices, center, and foci, and the eccentricity of the ellipse.

**Solution:**

$$\frac{16(x + 3/4)^2}{105} + \frac{8(y + 1)^2}{21} = 1$$

$$Z(-3/4, -1) \quad \text{Horizontal Major } \frac{\sqrt{105}}{4}$$

$$M(-3/4, -1 \pm \sqrt{42}/4) \quad V(-3/4 \pm \sqrt{105}/4, -1), \quad F(-3/4 \pm \sqrt{7}/4, -1)$$

$$e = \sqrt{15}/5 \cong .77459$$

**Problem Bonus** Find the 6th roots of -144.

**Solution:** Find  $\|\omega\|$  and  $\theta$

$$\|\omega\| = 144 \quad \theta = \pi$$

$$z_k = \sqrt[6]{144} \operatorname{cis} \left( \frac{\pi + 2\pi k}{6} \right) \quad k = 0, 1, 2, 3, 4, 5$$