MATH 2641: Linear Algebra I

Summer 2022 CRN: 55755-010

Instructor:	Dr. Joshua Miller	**Time:**	TTh 10:55-1:25
Review:	True-False	**Place:**	

1.1

1. (T/F) Every elementary row operation is reversible.
2. (T/F) Elementary row operations on an augmented matrix never change the solution set of the associated linear system.
3. (T/F) A 5×6 matrix has six rows.
4. (T/F) Two matrices are row equivalent if they have the same number of rows.
5. (T/F) The solution set of a linear system involving variables $x_1, \ldots, x_n$ is a list of numbers $(s_1, \ldots, s_n)$ that makes each equation in the system a true statement when the values $s_1, \ldots, s_n$ are substituted for $x_1, \ldots, x_n$ respectively.
6. (T/F) An inconsistent system has more than one solutions.
7. (T/F) Two fundamental questions about a linear system involve existence and uniqueness.
8. (T/F) Two linear systems are equivalent if they have the same solution set.

1.2

1. (T/F) In some cases, a matrix may be row reduced to more than one matrix in reduced echelon form, using different sequences of row operations.
2. (T/F) The echelon form of a matrix is unique.
3. (T/F) The row reduction algorithm applies only to augmented matrices for a linear system.
4. (T/F) The pivot positions in a matrix depend on whether row interchanges are used in the row reduction process.
5. (T/F) A basic variable in a linear system is a variable that corresponds to a pivot column in the coefficient matrix.
6. (T/F) Find a parametric description of the solution set of a linear system is the same as solving the system.
7. (T/F) Whenever a system has free variables, the solution set contains a unique solution.
8. (T/F) If one row in an echelon form of an augmented matrix is [0 0 0 0 3], then the associated linear system is inconsistent.
9. (T/F) A general solution of a system is an explicit description of all solutions of the system.

1.3

1. (T/F) The vector $\mathbf{u}$ results when a vector $\mathbf{u} - \mathbf{v}$ is added to the vector $\mathbf{v}$.
2. (T/F) An example of a linear combination of vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ is $\frac{1}{2}\mathbf{v}_1$.
3. (T/F) The weights $c_1, \ldots, c_p$ in a linear combination $c_1\mathbf{v}_1 + \cdots + c_p\mathbf{v}_p$ cannot all be zero.
4. (T/F) The solution set of the linear system whose augmented matrix is $[a_1\ a_2\ a_3\ b]$ is the same as the solution set of the equation $x_1\mathbf{a}_1 + x_2\mathbf{a}_2 + x_3\mathbf{a}_3 = \mathbf{b}$.
5. (T/F) When $\mathbf{u}$ and $\mathbf{v}$ are nonzero vectors, then span$\{\mathbf{u}, \mathbf{v}\}$ contains a line through $\mathbf{u}$ and the origin.
6. (T/F) The set span$\{\mathbf{u}, \mathbf{v}\}$ is always visualized as a plane through the origin.
7. (T/F) Asking whether the linear system corresponding to an augmented matrix $[a_1\ a_2\ a_3\ b]$ has a solution amounts to asking whether $\mathbf{b}$ is in span of $\mathbf{a}_1$, $\mathbf{a}_2$, and $\mathbf{a}_3$.

1.4

1. (T/F) The equation $A\mathbf{x} = \mathbf{b}$ is referred to as a vector equation.
2. (T/F) Every matrix equation $A\mathbf{x} = \mathbf{b}$ corresponds to a vector equation with the same solution set.
3. (T/F) If the equation $A\mathbf{x} = \mathbf{b}$ is inconsistent, the $\mathbf{b}$ is not in the set spanned by the columns of A.
4. (T/F) The equation $A\mathbf{x} = \mathbf{b}$ is consistent if the augmented matrix $[\ A\ \mathbf{b}]$ has a pivot position in every row.
5. (T/F) If A is an $m \times n$ matrix whose columns do not span $\mathbb{R}^m$, then the equation $A\mathbf{x} = \mathbf{b}$ is inconsistent for some $\mathbf{b}$ in $\mathbb{R}^m$.
6. (T/F) If the coefficient matrix $[A\ \mathbf{b}]$ has a pivot position in every row, then the equation $A\mathbf{x} = \mathbf{b}$ is inconsistent.

1.5

1. (T/F) A homogeneous system is always consistent.
2. (T/F) If $\mathbf{x}$ is a nontrivial solution of $A\mathbf{x} = \mathbf{0}$, then every entry of $\mathbf{x}$ is nonzero.
3. (T/F) The homogeneous equation has the trivial solution if and only if the equation has at least one free variable.
4. (T/F) The equation $A\mathbf{x} = \mathbf{b}$ is homogeneous if the zero vector is the solution.
5. (T/F) The equation $\mathbf{x} = \mathbf{p} + t\mathbf{v}$ describes a line through $\mathbf{v}$ parallel to $\mathbf{p}$.

1.7

1. (T/F) The columns of A are linearly independent if and only if $A\mathbf{x} = \mathbf{0}$ has the trivial solution.
2. (T/F) Two vectors are linearly dependent if and only if they lie on a line through the origin.
3. (T/F) If S is a linearly dependent set, then each vector is a linear combination of the other vectors in S.

4. (T/F) If a set contains fewer vectors than there are entries in the vectors, then the set is linearly independent.

5. (T/F) If $\mathbf{x}$ and $\mathbf{y}$ are linearly independent, and if $\{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$ is linearly dependent, the $\mathbf{z}$ is in span$\{\mathbf{x}, \mathbf{y}\}$.

1.8

1. (T/F) Every matrix transformation is a linear transformation.

2. (T/F) The codomain of the transformation $\mathbf{x} \mapsto A\mathbf{x}$ is the set of all linear combinations of the columns of A.

3. (T/F) Every linear transformation if a matrix transformation.

1.9

1. (T/F) A mapping $T : \mathbb{R}^n \to \mathbb{R}^m$ is onto $\mathbb{R}^m$ if every vector $\mathbf{x}$ in $\mathbb{R}^n$ maps onto some vector in $\mathbb{R}^m$.

2. (T/F) A transformation $T : \mathbb{R}^n \to \mathbb{R}^n$ is one-to-one if each vector in $\mathbb{R}^n$ maps onto a unique vector in $\mathbb{R}^m$.

3. (T/F) A is a 3×2 matrix, then the transformation $\mathbf{x} \mapsto A\mathbf{x}$ cannot map $\mathbb{R}^2$ onto $\mathbb{R}^3$.

4. (T/F) A is a 3×2 matrix, then the transformation $\mathbf{x} \mapsto A\mathbf{x}$ cannot be one-to-one.

2.1

1. (T/F) Each column of AB is a linear combination of the columns of B using weights from the corresponding column of A.

2. (T/F) $AB + AC = A(B + C)$

3. (T/F) $A^T + B^T = (A + B)^T$

4. (T/F) $(AB)^T = A^T B^T$

2.2

1. (T/F) In order for a matrix B to be the inverse of A, both equations $AB = I$ and $BA = I$ must be true.

2. (T/F) If A and B are $n \times n$ matrices, then $A^{-1}B^{-1}$ is the inverse of AB.

3. (T/F) If A is an invertible, then the inverse of A^{-1} is A itself.

4. (T/F) $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$.

5. (T/F) $(A^T)^{-1} = (A^{-1})^T$.

6. (T/F) If A is an invertible $n \times n$ matrix, then then the equation $A\mathbf{x} = \mathbf{b}$ is consistent for each $\mathbf{b}$ in $\mathbb{R}^n$.

7. (T/F) If A can be row reduced to the identity matrix, then A must be invertible.

8. (T/F) Each elementary matrix is invertible.

2.3

1. (T/F) If $A\mathbf{x} = \mathbf{0}$ has a nontrivial solution, then A is row equivalent to the $n \times n$ identity matrix.
2. (T/F) If the columns of A span $\mathbb{R}^n$, then the columns of A are linearly independent.
3. (T/F) If there is an $n \times n$ matrix D such that $AD = I$, then there is also an $n \times n$ matrix C such that $CA = I$.
4. (T/F) If $A\mathbf{x} = \mathbf{0}$ has a nontrivial solution, then A has exactly n pivots.
5. (T/F) If the columns of A are not linearly independent, then A is invertible.
6. (T/F) If A^T is not invertible, then A can still be invertible.

2.4

1. (T/F) If A is a block diagonal matrix, then A is invertible when most of the blocks are invertible.

2.8

1. (T/F) A subspace of $\mathbb{R}^n$ is any set H that is closed under addition.
2. (T/F) The column space space of a matrix A is the set of solutions to $A\mathbf{x} = \mathbf{b}$.
3. (T/F) Row operations affect the linear dependence relation among the columns of a matrix.
4. (T/F) The null space of an $m \times n$ matrix is a subspace of $\mathbb{R}^n$.
5. (T/F) The span of a set of vectors is not a subspace.

2.9

1. (T/F) If $\mathcal{B} = \{b_1, \ldots, b_n\}$ is a basis for a subspace H and $\mathbf{x} = c_1\mathbf{b}_1 + \cdots + c_n\mathbf{b}_n$, then the weights $c_1, \ldots, c_n$ are the coordinates of $\mathbf{x}$ relative to $\mathcal{B}$.
2. (T/F) Every line in $\mathbb{R}^n$ is not a subspace of $\mathbb{R}^n$.
3. (T/F) The dimension of the null space of a matrix A is the number of basic variables to the equation $A\mathbf{x} = \mathbf{0}$.
4. (T/F) The dimension of the column space of a matrix A is the rank of A.
5. (T/F) If you have p linearly independent vectors in p-dimensional vector space, then they form a basis.

3.2

1. (T/F) A row replacement affects the determinant of a matrix.
2. (T/F) If the columns of A are linearly independent, then the determinant is zero.
3. (T/F) If A is a block diagonal matrix, then the determinant is the product of all the determinants of each block.
4. (T/F) $\det A^{-1} = -\det A$.

5. (T/F) $\det(A+B) = \det A + \det B$.
6. (T/F) The determinant of the matrix ABC is not the product of the determinants of A, B, and C.
7. (T/F) Row interchange does not affect the determinant of a matrix.

4.1

1. (T/F) A vector is any element of a vector space.
2. (T/F) A subset H of a vector space V is a subspace if it satisfies the condition that the zero of V is in H.
3. (T/F) $\mathbb{R}^2$ is a subspace of $\mathbb{R}^3$.
4. (T/F) The polynomials of degree at most n are not a vector space.
5. (T/F) The set of matrices, is a vector space.
6. (T/F) A subspace is not a vector space.

4.2

1. (T/F) The null space is not a vector space.
2. (T/F) The column space is a vector space.
3. (T/F) For a matrix A, Row A = Col A^T.
4. (T/F) Col A is set of all solutions to $A\mathbf{x} = \mathbf{b}$.
5. (T/F) The kernel of a linear transformation is not a vector space.
6. (T/F) The range of a linear transformation is not a vector space.

4.3

1. (T/F) A single vector is always linearly independent.
2. (T/F) If a set S of nonzero vectors spans a vector space, then some subset of S is a basis.
3. (T/F) A basis only needs to span the space.
4. (T/F) A basis is the largest spanning set possible.
5. (T/F) The pivot columns of a matrix A is the basis vectors for Col A.
6. (T/F) The rows of a matrix A are the basis vectors for row A.

4.4

1. (T/F) If $\mathbf{x}$ is in $\mathbb{R}^n$ and if $\mathcal{B}$ contains n vectors, then the $\mathcal{B}$-coordinate vector of $\mathbf{x}$ is in $\mathbb{R}^n$.
2. (T/F) If P_B is the change of basis matrix, then $[\mathbf{x}]_{\mathcal{B}} = P_{\mathcal{B}}^{-1}\mathbf{x}$.
3. (T/F) If $\{\mathbf{v}_1, \ldots, \mathbf{v}_n\}$ is a linearly independent set, then $\{[\mathbf{v}_1]_{\mathcal{B}}, \ldots, [\mathbf{v}_n]_{\mathcal{B}}\}$ may not be linearly independent for a basis $\mathcal{B}$.

4.5

1. (T/F) If there exists a set of n elements that spans a space V, then $\dim V \geq n$.
2. (T/F) If every set of p elements in V fails to span V, then $\dim V > p$.
3. (T/F) If $p \geq 2$, and $\dim V = p$, then every set of $p - 1$ nonzero vectors is linearly independent.
4. (T/F) All vector spaces are finite dimensional.

4.6

1. (T/F) The columns of a change of coordinates matrix are linearly independent.
2. (T/F) The columns of the change of coordinates matrix, from $\mathcal{B}$ to $\mathcal{C}$, are the $\mathcal{B}$-coordinate vectors of the vectors in $\mathcal{C}$.
3. (T/F) The change of coordinates matrices are invertible and allow us to do the change of coordinates in the other direction.

5.1

1. (T/F) If $A\vec{x} = \lambda\vec{x}$ for some vector $\vec{x}$, then λ is an eigenvalue of A.
2. (T/F) If $A\vec{x} = \lambda\vec{x}$ for some scalar λ, then $\vec{x}$ is an eigenvector of A.
3. (T/F) If 0 is an eigenvalue, then A is invertible.
4. (T/F) If A is a triangular matrix, then the eigenvalues are the entries of the first row.
5. (T/F) An eigenspace of A is a null space of a certain matrix.
6. (T/F) A number c is an eigenvalue of A if and only if the equation $(A - cI)\vec{x} = \vec{0}$ has a nontrivial solution.
7. (T/F) If v_1 and v_2 are linearly independent, then the eigenvalues are the same.
8. (T/F) There is only one eigenvector for each eigenvalue.

5.2

1. (T/F) The matrix A and A^T have different sets of eigenvalues.
2. (T/F) The matrices A and $B^{-1}AB$ have the same sets of eigenvalues for every invertible matrix B.
3. (T/F) If two matrices are similar, then they have different eigenvalues.
4. (T/F) $\det(A - \lambda I)$ is the characteristic equation of the matrix A.
5. (T/F) If two matrices have the same eigenvalues, then they are similar.
6. (T/F) A matrix can have more eigenvalues than rows.

5.3

1. (T/F) If A is invertible, then it is diagonalizable.
2. (T/F) If $AP = PD$, with D diagonal, then the nonzero columns of P are eigenvectors of A.
3. (T/F) A is diagonalizable if there are fewer than n eigenvectors.
4. (T/F) If $\mathbb{R}^n$ has a basis of eigenvectors of A, then A is not diagonalizable.

5.4

1. (T/F) Similar matrices have the same eigenvalues and eigenvectors.
2. (T/F) Only linear transformation on finite vector spaces have eigenvectors.
3. (T/F) If there is a nonzero vector in the kernel of the linear transformation T, then 0 is an eigenvalue of T.
4. (T/F) The bird is the word.