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MATH 1001
Quantitative Reasoning

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The purpose of these notes is to provide a concise overview of the material covered in MATH 1001 at Georgia State University. Note that the chapters are presented in order that they are presented in the course due to exam scheduling.

Chapter 1

1.1

While studying mathematics there are two types of reasoning we want to consider. The first type is called **inductive reasoning** which can be described as

Definition 1 (Inductive Reasoning)**.** *Arriving at a general conclusion through observation of specific examples.*

Even though this type of reasoning is a good way of drawing conclusions, it might be that these conclusions are false. For this reason, it is common for these conclusions to be called **hypotheses**. Since these conclusions are nothing more than educated guesses, any example that disproves it is called a **counterexample**.

An example of non-mathematical inductive reasoning goes as follows: *I pull three coins from a jar, all three are quarters. Therefore all the coins in the jar are quarters.* In this example we see that we are drawing a conclusion after only pulling three coins. We do not know if there are non-quarter coins in the jar without pulling the remainder of the coins out.

Another example of inductive reasoning that is false goes as follows: *My wife is a mother and she has blonde hair, therefore all mothers have blonde hair.* This example is clearly false considering we known many different mothers with hair that is not blonde. For this example, Halle Berry would be a counter-example.

Other ways we can look at inductive reasoning is through sequences and visual patterns where we try to figure out the next term in the sequence by finding a pattern. Common patterns to look for are multiplication or addition if the sequence is increasing and division or subtraction if the sequence is decreasing. Let's look at a number sequence.

Example 2. *Find the next term in the sequence*

$$1, 6, 11, 16, 21, 26, \cdots$$

First we need to find a pattern, and the first way to try and find one is through checking our standard arithmetic, addition, subtraction, multiplication, and division. Since our sequence is increasing, it has to be either addition or multiplication, so lets look at the differences between successive terms.

6-1=5
11-6=5
16-11=5
21-16=5
26-21=5

Here we see that the difference between successive terms is five, so inductive reasoning says that the next term is 26+5=31.

Example 3. *Find the next term in the sequence*

$$64, 16, 4, 1, \frac{1}{4}, \frac{1}{16}, \frac{1}{64}, \cdots$$

First we need to find a pattern, and the first way to try and find one is through checking our standard arithmetic, addition, subtraction, multiplication, and division. Since our sequence is decreasing, it has to be either subtraction or division, so lets look at the differences between successive terms.

64-16=48
16-4=12
4-1=3
1-$\frac{1}{4}$=.75

Here we see that the difference between successive terms is not the same so it cannot be subtraction. Maybe it is division, and one way to check that is by looking at the ratios of the terms.

$$\frac{64}{16} = 4$$

$$\frac{16}{4} = 4$$

$$\frac{4}{1} = 4$$

$$\frac{1}{1/4} = 4$$

We just found out that the ratio of successive terms is four and is decreasing, so the pattern is dividing by 4. Thus by inductive reasoning the next term is

$$\frac{\frac{1}{64}}{4} = \frac{1}{64 \cdot 4} = \frac{1}{256}.$$

Sometimes however, none of the standard arithmetic will yield a pattern and will require more complicated steps to find the next term in the sequence. Consider the following example.

Example 4. *Find the next term in the sequence*

$$2, 6, 5, 15, 14, 42, 41, 123, \cdots$$

First we need to find some kind of pattern and unfortunately this sequence does follow one operation strictly. Let us check the some of the pairs:

$$2 \times 3 = 6$$
$$5 \times 3 = 15$$
$$14 \times 3 = 42$$
$$41 \times 3 = 123$$

So for first and second, third and forth, fifth and six, and so, does follow a pattern namely multiplication by three to get the next term. However, if we look at the pairs, 6 and 5, 15 and 14, and 42 and 41 all differ by 1. There is the pattern! Since $41 \times 3 = 123$*, then the term after 123 would be 122.*

The other type of reasoning is called **deductive reasoning** which can be described as

Definition 5 (Deductive Reasoning)**.** *The process of proving a specific conclusion from general statements.*

The strength of this reasoning is that uses general statements to arrive at a conclusion instead of logic alone. On the other hand, the weakness of deductive reasoning comes from when the general statements or premises we start with are incorrect. Any conclusion that is proven by deductive reasoning is called a **theorem**.

Let us consider the difference between inductive and deductive reasoning by considering the following example.

Example 6. *Consider the procedure where we select a number. Add 3. Double the sum, and add 4. Then divide that number by 2 and subtract the original selected number.*

We will pick a few numbers and try to find a pattern, this is inductive reasoning.

1) Select n	$n = 2$	$n = 3$	$n = 5$
2) Add 3	*2+3=5*	*3+3=6*	*5+3=8*
3) Double result	$5 \cdot 2 = 10$	$6 \cdot 2 = 12$	$8 \cdot 2 = 16$
4) Add 4	*10+4=14*	*12+4=16*	*16+4=20*
5) Divide by 2	$14/2 = 7$	$16/2 = 8$	$20/2 = 10$
6) Subtract original	*7-2=5*	*8-3=5*	*10-5=5*

Based on these examples we can use inductive reasoning to say that this sequence of steps returns the same number we started off with. This does not prove anything though! Now let n *be whatever it wants.*

1) Select n	n
2) Add 3	$n+3$
3) Double result	$2(n+3) = 2n+6$
4) Add 4	$2n+6+4 = 2n+10$
5) Divide by 2	$(2n+10)/2 = n+5$
6) Subtract original	$n+5-5 = n$

These steps proved that if we follow the above steps for any n, we will always end with n as well. This process of proving it for general n is deductive reasoning.

1.2

In this section the main idea is **estimation** which is the process of getting an approximate answer to a question. For the purpose of this course we will look at estimation using whole number rounding.

The whole numbers are $\mathbb{W} = \{0, 1, 2, 3, ...\}$, and the key to whole number rounding is the digits 0,1,2,...,9 and how their placement in a number determines the value. The position determines the value of the digit, for example consider the following number $8, 675, 309$

8	millions
6	hundred thousands
7	ten thousands
5	thousands
3	hundreds
0	tens
9	one

This pattern can continue on to the billions, trillions, and so on. Further, this pattern works for decimals except it starts with the tenths spot, i.e. .23 is 2 tenths and 3 hundredths. To round with whole numbers we follow two steps:

1. Look at the digit to the right of the rounding spot.
2. a) If the digit to the right is 5 or larger, then add one to the digit being rounded and replace all digits to the right with zeros.
   b) If the digit is less than 5, then do nothing to the digit being rounding and replace all the digits to the right with zeros.

Example 7. *Round the number $2, 678, 932, 123$ to the nearest hundred-millions and the nearest ten. The first thing we want to do is identify the digit being rounded which is located in red*

$$2, 678, 932, 123.$$

This number is a 6 and the number immediately to the right, in blue, is a 7. Since 7 is larger than 5, then we add one to the 6 and replace the digits to the right with zeros

$$2, 700, 000, 000.$$

Following a similar method for rounded to the nearest tens we see

$$2, 678, 932, 123.$$

Since 3 is less than 5, then we leave the 2 alone and replace the 3 with a zero

$$2, 678, 932, 120.$$

Example 8 (Application)**.** *A librarian makes $\$23$ per hour. Estimate their weekly and annual income if they work full time each week, meaning 40 hours per week.*

First, we can round the pay rate of the librarian, which would round down to $\$20$ per hour. Then since weekly pay rate is calculated by multiplying your per hour rate times then number of hours worked in a week we have that

$$\$20 \textit{ per hour} \times 40 \textit{ hours per week} = \$800 \textit{ per week}.$$

To find the approximate annual salary one is able to a simply approximation by rounding 52 weeks to 50 weeks and using the same formula as follows

$$\$800 \textit{ per week } \times 50 \textit{ weeks per year} = \$40,000 \textit{ per year} .$$

The last type of estimation we will discuss, since the rest involve visual approximation of a graph, revolves are circle graphs. First, we need to talk about **percents** which are ways to represent numbers as a part of 100. The two big parts to percents remember are

1. Converting from percent form to decimal form by moving the decimal two digits to the left and dropping the % sign, i.e. $41\% \Rightarrow .41$.

2. A is P percent of B.

According to the U.S. Census Bureau, in 2018, the 218,465,624 Americans 25 years and older were surveyed. After being asked for their definition of old age, the results are as follows, 32%- declining mental function, 32%- declining physical function, 28% reaching a specific age number, and 8%- becoming grandparents.

1. Determine a calculation that calculates how many 25 years and older Americans define old age by reaching a specific age.

2. Use rounding to give a reasonable estimation.

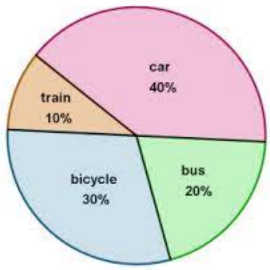
For the first question, we want to use A is P percent of B, where A is the number that defines old age by a number, P is 28% and B is the total number surveyed. Therefore the calculation is given by

$$\text{Define old age by a number } = .28 \times (218,465,624).$$

Note that we changed our percent to the decimal form. For the second part, we want to round and the two numbers we can round here are $.28$ and $218,465,624$. The percent rounds up to $.30$ and a "reasonable" for the number surveyed would be to the nearest ten-million (reasonable is subjective here). Thus the approximation is

$$A = (.3) \times (220,000,000) = 66,000,000.$$

Example 9. *The following chart shows the percentage of people in Titusville ISD who commute to school in the following manners: bicycle, bus, car, or train. Assume there are 154,324 people that commute to school. Determine a calculation that calculates how many people commute to school by train or bus. Further, use rounding to give a reasonable estimation.*



Looking at the chart we see that there 10+20=30% of the commuters ride the train or bus. Thus the formula to find the number of commuters that ride the train or bus is given by

$$\textit{Bus or Train Riders } = .3 \times (154,324).$$

The only type of rounding we can do for this problem is to round the number of commuters. In this case a "reasonable"number to round to is the ten-thousand spot, so 154,324 would round to 150,000. Therefore the approximation of the number of commuters using the train or bus is

$$(.3) \times (150,000) = 45,000.$$

The remaining topics covered in 1.2 are about estimation using line graphs which may be added at a later time.