Exam 3, Nov. 2018

1) Find the domain of the function f(x)=log(7x+1)

Note: We cannot take the log of negative numbers or zero.

Solve what is inside the log for zero and use number line test.

$7x+1=0$

$x=-1/7$

$x=-2$

$-14+1<0$

$-$

$x=0$

1

$+$

$-1/7$

Domain $(-1/7, \infty)$.

2) Solve: $1+4e^x=13$ Leave answers exact.

Note: Logs and exponentials are inverses of each other and are used to solve equations involving each other.

$\log_b a = c \iff b^c = a$

$1+4e^x=13$

$4e^x=12$

$\frac{4e^x}{4}=\frac{12}{4}$

$e^x=3$

$\ln(e^x)=\ln 3 \implies x=\ln 3$

get to exp form

$b=e$, $a=3$, $c=x$

$\log_e 3 = x$

$\ln 3 = x$

3) Using the values of log(a)=6 and log(b)=4, find $\log\left(\frac{b}{\sqrt{a}}\right)$

Note: We want to use log properties here

1) $\log A + \log B = \log AB$

2) $\log A - \log B = \log(A/B)$

3) $\log(A^r) = r\log A$

$x^{1/n} = \sqrt[n]{x}$

Expand first, terms of $\log a$ + $\log b$

$\overset{(2)}{=} \log b - \log(\sqrt{a}) = \log b - \log(a^{1/2})$

$\overset{(3)}{=} \log b - \frac{1}{2}\log a = 4 - \frac{1}{2}(6) = 4 - 3 = 1$

4) Solve: 3+ $\log_4(x-1)=6$. Leave answers exact.

Note: Similar to question two, we want to use the fact that log and exponentials are inverses

of each other.

Recall the relation from earlier.

$\log_b a = c \iff b^c = a$

Want to have $\log_b a = c$

$3 + \log_4(x-1) = 6$

$\log_4(x-1) = 3$

$b = 4,\ a = (x-1)$

$c = 3$

$\Rightarrow 4^3 = x-1$

$64 = x-1$

$\Rightarrow x = 65.$

5) Rewrite using the base b via the change of base formula.

1) $\log_A(W) \overset{(4)}{=} \dfrac{\log_b W}{\log_b A}$

2) Condense into a single logarithmic expression using the properties of logarithms.

$\log_5(z) + \log_{25}(z+1)$

$\overset{(4)}{=} \log_5 z + \dfrac{\log_5(z+1)}{\log_5 25} = \log_5 z + \frac{1}{2}\log_5(z+1) \overset{(3)}{=}$

$= \log_5 z + \log_5\left(\sqrt{z+1}\right)$

$\overset{(1)}{=} \log_5\left(z\sqrt{z+1}\right)$

Recall: Our log properties

1) $\log A + \log B = \log(AB)$

2) $\log A - \log B = \log(A/B)$

3) $\log(A^r) = r\log A$

4) $\log_b A = \dfrac{\log_c A}{\log_c b}$

Note: it does not matter what base we change to.

$\log_5 25 \Rightarrow 5^2 = 25$

6) Find the domain of the function $f(x)= \sqrt{x^2+2x}$.

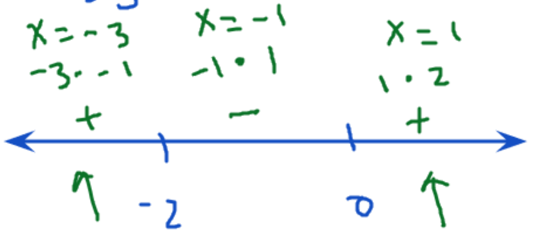
Note: For even power roots, we cannot have negatives in our domain. $\sqrt{\ }$, $\sqrt[4]{\ }$

For odd power roots, it does not matter. Domain is all real numbers.

$\sqrt[3]{\ }, \sqrt[5]{\ }, \ldots$

Solve $x^2+2x=0$

$x(x+2)=0$



Domain $(-\infty, -2] \cup [0, \infty)$

7) Solve and check your answer: $\frac{3}{2x+1} = \frac{4}{x-3}$

Get rid of fractions $3(x-3) = 4(2x+1)$

cross multiply $3x - 9 = 8x + 4$

Combine terms $-9 = 5x + 4$

$-13 = 5x$

$x = -13/5$

$$\frac{3}{2\left(\frac{-13}{5}\right)+1} = \frac{3}{\frac{-24}{5}+\frac{5}{5}} = \frac{3}{\frac{-21}{5}} = \frac{-15}{21} = -5/7$$

$$\frac{4}{\frac{-13}{5}-3} = \frac{4}{\frac{-13}{5}-\frac{15}{5}} = \frac{4}{\frac{-28}{5}} = \frac{-20}{28} = -5/7 \checkmark$$

8) Solve the rational inequality: $\frac{3x-3}{x+5} > 0$ Be sure to justify your answer.

Recall: For rational functions,
zeros of the numerator are zeros of the rational function

zeros of the denominator are poles of the rational function.

$(-\infty, -5) \cup (1, \infty)$

→ For >,< zeros and poles of the rational function use (or)

For ≥,≤ zeros of the rational function use [or] while poles use (or)

1) Find zeros & poles

zeros $3x - 3 = 0 \Rightarrow x = 1$

poles $x + 5 = 0 \Rightarrow x = -5$

2) Justification: number line test

$x = -10$ $(-)/(-)$ | $x = 0$ $-3/5$ | $x = 2$ $3/7$

$+$ ↑ | -5 | $-$ | 1 | $+$ ↑

9) The supply function for a certain product is given by $p=200(3^q)$, where p is the price of the product and q is the quantity supplied at that price. If the price of the product is $16,200, how many units are will be supplied?

$p = 16{,}200$

find q.

$$\frac{16200}{200} = \frac{200(3^q)}{200}$$

$$81 = 3^q$$

$$\log_3 81 = \log_3 3^q$$

$$\log_b a = c \iff b^c = a$$

$$b = 3,\ c = q,\ a = 81$$

$$(3) \Rightarrow \log_3 81 = q \log_3 3 = q \Rightarrow q = 4$$

$\log_3 81 = 4$

$81 = 3^q$

$9^2 = 3^q$

$(3^2)^2 = 3^q \rightarrow 3^4 = 3^q$

$q = 4$

10) a) Find the inverse when f(x)=-5x+1.

(1) $y = -5x + 1$

2) $x = -5y + 1$

$$\frac{x-1}{-5} = \frac{-5y}{-5}$$

$$y = \frac{x-1}{-5}$$

$$f^{-1}(x) = \frac{x-1}{-5}$$

Finding inverses.

1) Replace f(x) or g(x) with y

2) Flip x and y in the equation.

3) Resolve for y, this is our inverse

b) Find the inverse when $g(x)= \log_2 (x+1)$.

(1) $y = \log_2(x+1)$

(2) $\underline{x} = \log_{\underline{2}}(y+1)$

$2^x = y + 1$

$2^x - 1 = y$

$$g^{-1}(x) = 2^x - 1$$

11) Suppose R(t)=2t+1 is a function that gives the radius of a circular oil spill at t minutes. Given $A(r)=\pi r^2$ find the expression for A(R(t)), and leave your final answer in terms of π

Recall: Function composition, for functions f(x) and g(x) we have that:

f(g(x)) is saying, "replace every x in f(x) with the ENTIRE function g(x)."

$$A(R(t)) = \pi(2t+1)^2$$

replace r in A(r) with R(t)

$$= \pi(4t^2+4t+1)$$

$$= 4\pi t^2 + 4\pi t + \pi$$

12) Given $r(x)= x^2 +6x-5$ and m(x)=2x-1, find m(r(x)) and write your answer in the form $ax^2 +bx+c$.

$$m(r(x)) = 2(x^2+6x-5)-1$$

$$= 2x^2 + 12x - 10 - 1$$

$$= 2x^2 + 12x - 11.$$

13) Graph $f(x)= 3^x$ and $g(x)= \log_3(x)$. Include at least 4 points on the graph.

Recall:

Basic Shape of exponentials is ↷ , a^x

Basic Shape of logs is ↷ , $\log_a x$

Find Points first:

$f(0) = 3^0 = \underline{1}$

$f(\underline{1}) = 3^1 = \underline{3}$

$f(\underline{2}) = 3^2 = \underline{9}$

$f(\underline{-1}) = 3^{-1} = \underline{1/3}$

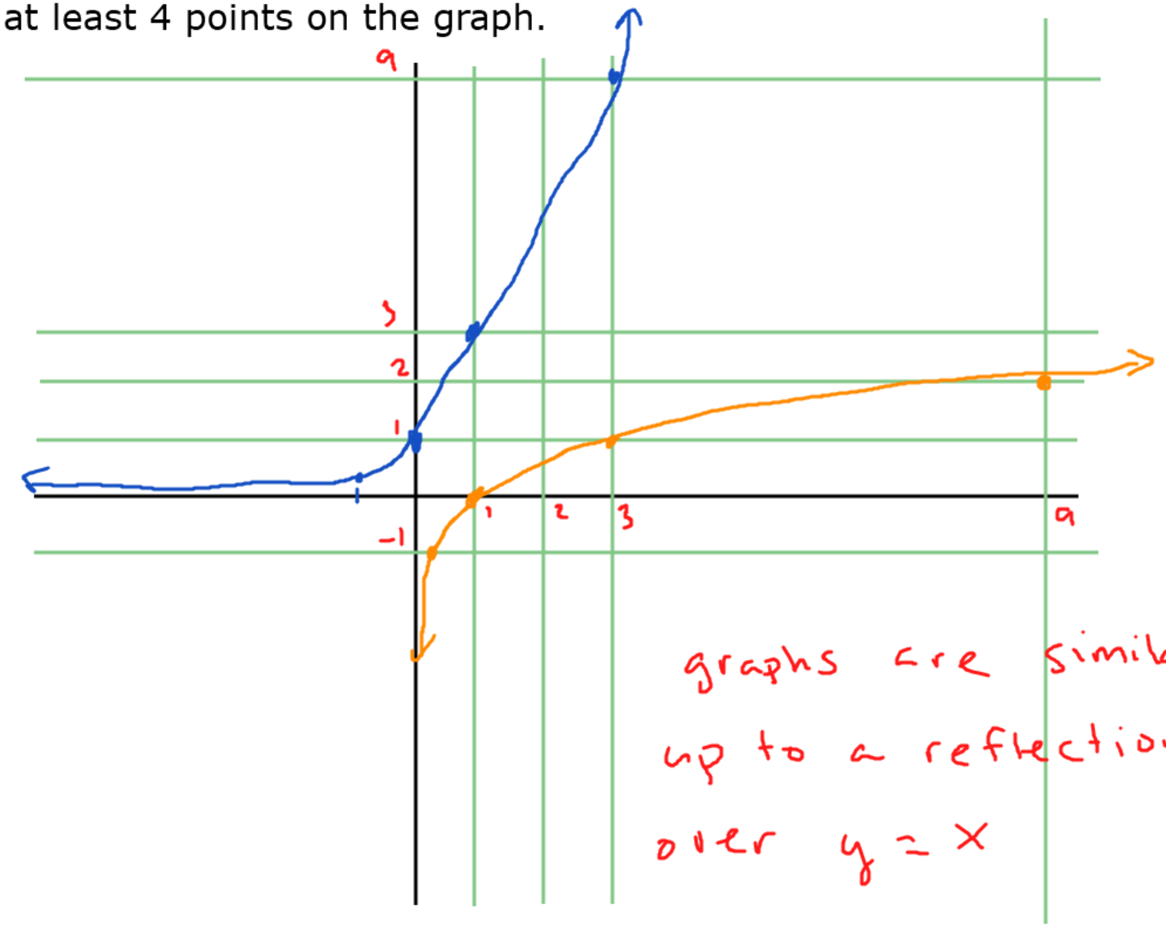
$g(0)$ DNE

$g(\underline{1}) = \log_3(1) = \underline{0}$

$g(\underline{3}) = \log_3 3 = \underline{1}$

$g(\underline{9}) = \log_3 9 = \underline{2}$

$g(\underline{1/3}) = \log_3(1/3) = \underline{-1}$



graphs are similar up to a reflection over $y = x$

Note symmetry, they are inverses

Extra) Find the value M such that x=2, $\frac{7}{3x+22} = \frac{3}{x-M}$

Replace $x=2$ in our equation

$$\frac{7}{3(2)+22} = \frac{3}{2-M}$$

$$\frac{7}{28} = \frac{3}{2-M}$$

$$\frac{1}{4} = \frac{3}{2-M}$$, cross multiply

$$1(2-M) = 4(3)$$

$$2-M = 12$$

$$\boxed{-10 = M}$$

Extra Note) The change of base formula allows us to view logarithms in different ways!

$$\log_b a = \frac{\log_c a}{\log_c b}$$ c our new base

$$\log_9 42 \iff \frac{\log 42}{\log 9} \text{ (base 10)} \iff \frac{\log_3 42}{\log_3 9} = \frac{1}{2}\log_3 42 \text{ (base 3)}$$

$$\iff \frac{\ln 42}{\ln 9} \text{ (base e)}$$