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MATH 1001
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MATH 1001
Quantitative Reasoning

Fall 2021

The purpose of these notes is to provide a concise overview of the material covered in MATH 1001 at Georgia State University. Note that the chapters are presented in order that they are presented in the course due to exam scheduling.

Chapter 10

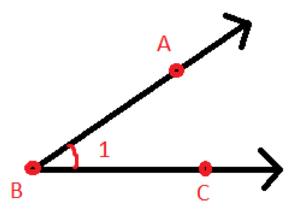
In chapter 10 we will be looking at some of the basics of geometry including angels, polygons, perimeters, and angles.

10.1

First we will develop the building tools of geometry by defining a few key objects. A **point**, represented by a dot, has no length, width, or thickness, i.e. zero dimensional. Going up one dimension we have a **line** which connects two points along the shortest path. Lines have no thickness and goes on for infinity in both directions and is“one-dimensional.Ïn "two dimensions,"we have a **plane** which is a flat surface with no thickness or boundary.

The typical notation for the line through the points A and B is $\overleftrightarrow{AB}$ or $\overleftrightarrow{BA}$. Further, we can ßplitöur lines to create **rays**, denoted $\overrightarrow{BA}$, which starts at the endpoint B and goes to infinity through the point A. The last type of "line"we care about is **line segment** which is the portion of the line joining the two points, including both endpoints, and is denoted $\overline{AB}$ or $\overline{BA}$.

An **angle**, $\measuredangle$, is formed by the union of two rays with a common endpoint, where one ray is called the initial side and the other is the terminal side. The common endpoint is called the vertex. To name an angle, there are several ways that are typically utilize the following names for the given angle:



$\measuredangle ABC$, $\measuredangle CBA$, $\measuredangle 1$, and if there is no confusion as to what angle we are referencing we can call it by the vertex $\measuredangle B$. Angles are measured in degrees, $^\circ$, and for one ray (side of the angle) to rotate back on itself, is 360°. Therefore

$$1^\circ = \frac{1}{360} \text{ of this rotation.}$$

For angles, there are some special angles that have specific names:

1. Right angle- quarter rotation or 90°
2. Acute angle- less than a quarter rotation, less than 90°
3. Obtuse angle- greater than quarter rotation, greater than 90°
4. Straight angle- half rotation, 180°

Definition 1. *Two angles are complementary if the sum of their angle measures sums to 90°. To find the complementary angle measure, subtract the given measure from 90°.*

Example 2. *Find $m\measuredangle DBC$ if $\measuredangle DBC$ and $\measuredangle ABD$ form a right angle and $m\measuredangle ABD = 62^\circ$.*
Since they are complementary angles, then

$$m\measuredangle DBC = 90^\circ - m\measuredangle ABD = 90 - 62 = 28^\circ.$$

Definition 3. *Two angles whose measures sum to 180° are called **supplementary** angles.*

Example 4. *Find $m\measuredangle DBC$ if $\measuredangle DBC$ and $\measuredangle ABD$ are supplementary angles and $m\measuredangle ABD$ is $66°$ greater than $m\measuredangle DBC$. Find both angle measures.*
Since they are supplementary angles, then

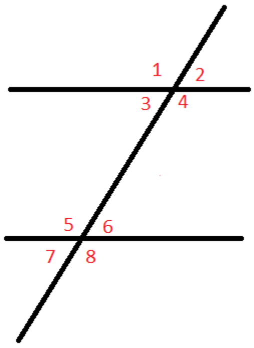
$$m\measuredangle DBC + m\measuredangle ABD = 180°.$$

Subbing in $m\measuredangle ABD = m\measuredangle DBC + 66$ implies that

$$2m\measuredangle DBC = 180 - 66 \qquad \Rightarrow \qquad m\measuredangle DBC = 57°.$$

Thus $m\measuredangle ABD = 57 + 66 = 123°$.

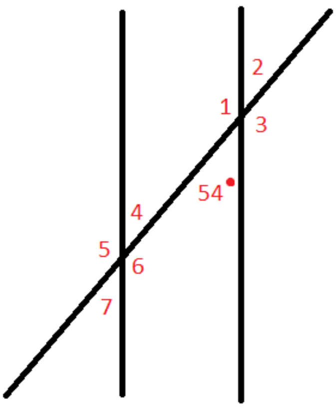
Next we will consider angles that are created from the intersection of lines, consider the following example where two parallel lines are intersected by a third, called a **transversal**. This intersection creates eight angles.



The following pairs of angles have the same measure! I will drop the angle symbol here for space concerns. Note that interior means the angles occur between the parallels, so angles 3, 4, 5, and 6, while exterior means outside the parallels, so angles 1, 2, 7, and 8.

1. Vertical angles- opposite angles formed when two lines intersect, they come in pairs. $1\&4$, $2\&3$, $5\&8$, and $6\&7$.
2. Alternate interior angles- no common vertex and on opposite sides of the transversal, they also come in pairs. $3\&6$ and $4\&5$.
3. Alternate exterior angles- no common vertex and on opposite sides of the transversal, they also come in pairs. $1\&8$ and $2\&7$.
4. Corresponding Angles- on the same side of the transversal and one is interior and the other is exterior, they also come in pairs. $1\&5$, $2\&6$, $3\&7$, and $4\&8$.

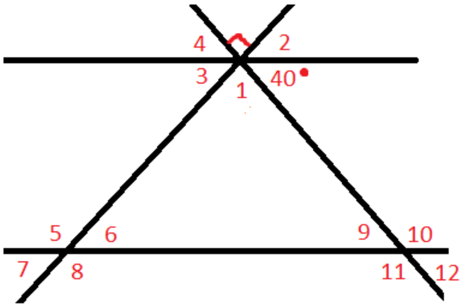
Example 5. *Find the missing angle measures*



1. *$\measuredangle 2 = 54°$ is it is a vertical angle with the given angle.*
2. *$\measuredangle 2 = \measuredangle 7 = 54°$ since they are alternate exterior.*
3. *$\measuredangle 7 = \measuredangle 4 = 54°$ since they are vertical angles.*

4. $\measuredangle 5 = \measuredangle 6 = \measuredangle 1 = \measuredangle 3 = 180 - 57 = 123°$ *since they are all supplementary to the angle* $57°$.

Example 6. *Find the missing angle measures*

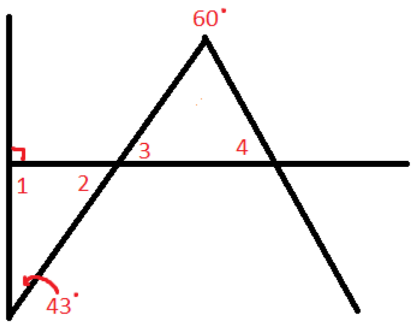


1. $\measuredangle 1 = 90°$ *since they are vertical angles.*
2. $\measuredangle 4 = 40°$ *since they are vertical angles with the given* $40°$.
3. $\measuredangle 3 = 180 - 90 - 40 = 50°$ *since angles 3, 4 and the given* $90°$ *are supplementary.*
4. $\measuredangle 3 = \measuredangle 2 = 50°$ *since they are vertical.*
5. $\measuredangle 3 = \measuredangle 7 = \measuredangle 6 = 50°$ *since they are corresponding angles between 7 and 3, and 7 and 6 are vertical.*
6. $\measuredangle 5 = \measuredangle 8 = 180 - 50 = 130°$ *since they are supplementary to the* $50°$ *angles.*
7. $\measuredangle 12 = \measuredangle 9 = 40°$ *since the given* $40°$ *and 12 are corresponding angles and 9 and 12 are vertical angles.*
8. $\measuredangle 10 = \measuredangle 11 = 180 - 40 = 140°$ *since they are supplementary to the* $40°$ *angles.*

10.2

In this section we will look at triangles, and the most important fact about triangles is the the sum of the angles add to $180°$. Thus we can consider some problems similar to the previous sections, but now armed with new information!

Example 7. *Find the remaining measures.*



1. $\measuredangle 1 = 90°$ *since it is complementary to the given right angle.*
2. $\measuredangle 2 = 180 - 43 - 90 = 47°$ *since angles 1,2, and the given* $43°$ *are the angles of a triangle.*
3. $\measuredangle 2 = \measuredangle 3 = 43°$ *since they are vertical angles.*
4. $\measuredangle 4 = 180 - 60 - 47 = 73°$ *since angles 3,4, and the given* $60°$ *are the angles of a triangle.*

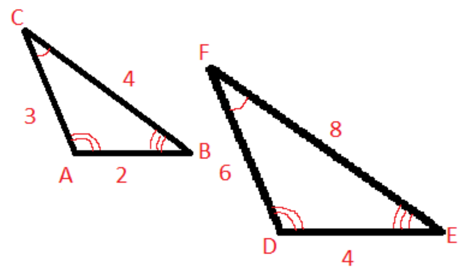
For triangles we are able to classify them by their angles and also their sides.

1. Acute triangle- all angles are acute
2. Right triangle- there is exactly one right angle
3. Obtuse triangle- there is exactly one obtuse angle
4. Isosceles triangle- two sides are equal length, this in turn forces two angles to have the same measure

5. Equilateral triangle- all sides have the same length, and therefore all angles are the same measure
6. Scalene triangle- no two sides are equal

Something we are interested in is when two triangles have the same shape, but not necessarily the same size. When this happens we call the shapes **similar**. For triangles, if two corresponding angles of one triangle are equal in measure to another triangle's angles, then they are similar.

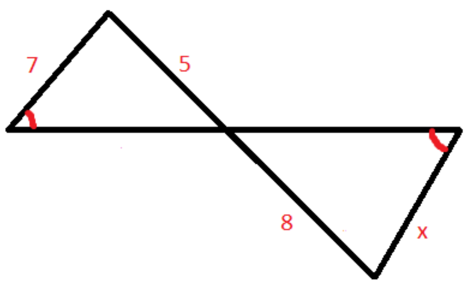
Another way to check to see if two triangles are similar is by checking the ratios of the corresponding sides. If they are similar, then all the ratios are the same. For example,



$$\frac{\overline{AC}}{\overline{DF}} = \frac{6}{3} = 2 \qquad \frac{\overline{CB}}{\overline{FE}} = \frac{8}{4} = 2 \qquad \frac{\overline{AB}}{\overline{DE}} = \frac{4}{2} = 2.$$

In actuality we can use these ratios to find missing side lengths for similar triangles.

Example 8. *Explain why these triangles are similar and find the value x.*



The triangles are similar since they have two corresponding angles that are the same measure, the given marked angles and the fact that there is a pair of vertical angles.

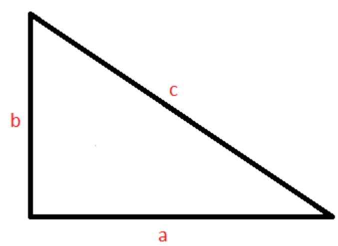
To find the the missing value x, we can use the ratios since the triangles are equal. One ratio will be made up of the sides opposite the marked angles, and the other will correspond to the sides opposite the vertical angles.

$$\frac{7}{x} = \frac{5}{8} \qquad \Rightarrow \qquad 5x = 56 \qquad \Rightarrow \qquad x = \frac{56}{5} \approx 11.2.$$

The main theorem for triangles, is for right triangles, and is known as **Pythagorean Theorem**. This is stated as follows

Theorem 9. *For any triangle with legs of length a, b, and hypotenuse of length c, then*

$$a^2 + b^2 = c^2$$



For questions involving Pythagorean theorem, two types can be asked. Utilizing the same triangle above.

Example 10. *Find the missing side of the right triangle with $a = 12$ and $b = 9$.*
For this triangle, we are missing the hypotenuse which is a direct application of the theorem. Therefore the hypotenuse is given by

$$c = \sqrt{a^2 + b^2} = \sqrt{12^2 + 9^2} = \sqrt{225} = 15.$$

Example 11. *Find the missing side of the right triangle with $a = 120$ and $c = 122$.*
For this triangle, we are missing a leg of the triangle, namely b. Therefore we need to solve for b first which is given by

$$b = \sqrt{c^2 - a^2} = \sqrt{122^2 - 120^2} = \sqrt{14,884 - 14,400} = \sqrt{484} = 22.$$

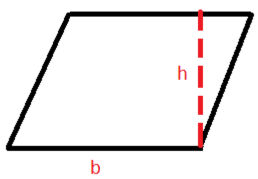
10.3 & 4

Definition 12. *A **polygon** is any closed shape in the plane formed by three or more line segments that intersect only at the endpoint.*

*A polygon is **regular** if all sides are the same length and angles all have the same measure.*

Definition 13. *The **perimeter** of a polygon is the sum of all lengths of the sides.*

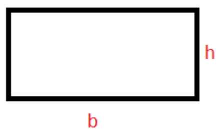
The regular polygon part is in progress, and is not too vital for the exam, please see the handwritten notes for pictures and names of these polygons for now. We will look at the area formulas for the following polygons, perimeter formulas will be given in some instances.



A **parallelogram** is a special type of quadrilateral (four sided polygons) where both pairs of opposite sides are parallel and have the same length. The formula for the **area of a parallelogram** is $A = bh$ where b and h are defined as above. A **rhombus** is a parallelogram where all sides are the same length.

Example 14. *Find the area of a parallelogram with $b = 8$ cm $h = 4$ cm.*

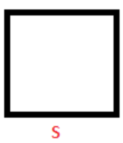
$$A = bh = 8 \times 4 = 32 \ cm^2.$$



A **rectangle** is a parallelogram with four right angles, opposite sides are parallel and have the same length. The **area of a rectangle** is $A = bh$ and the **perimeter** is $P = 2b + 2h$, where b and h are defined for the above figure.

Example 15. *Find the area and perimeter of a rectangle with $b = 5$ cm $h = 4$ cm.*

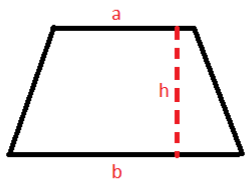
$$A = bh = 5 \times 4 = 20 \ cm^2,$$
$$P = 2b + 2h = 2 \times 5 + 2 \times 4 = 18 \ cm.$$



A **square** is a rectangle where all four sides are the same length. The **area of a square** is given by $A = s^2$ and the **perimeter** is given by $P = 4s$, for s defined above.

Example 16. *Find the area and perimeter of a square with $s = 5$ cm.*

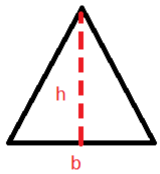
$$A = s^2 = 5^2 = 25\ cm^2,$$
$$P = 4s = 2 \times 5 = 20\ cm.$$



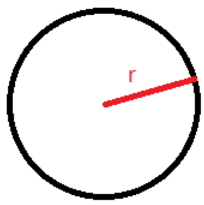
A **trapezoid** is a quadrilateral with exactly one pair of parallel sides, denoted a and b in the above figure. The **area of a trapezoid** is given by $A = \frac{1}{2}h(a + b)$.

Example 17. *Find the area of a trapezoid with $a = 32$ cm, $b = 46$ cm $h = 13$ cm.*

$$A = \frac{1}{2}h(a + b) = \frac{1}{2} \times (13)(32 + 46) = 507\ cm^2.$$



The **area of a triangle** is given by $A = \frac{1}{2}bh$.



Example 18. *Find the area of a triangle with $b = 5$ cm $h = 4$ cm.*

$$A = \frac{1}{2}bh = \frac{1}{2} \times 5 \times 4 = 10\ cm^2.$$

For a circle we have a few formulas, but let us formally define a circle first.

Definition 19. *A **circle** is a set of points in the plane equally distanced from a given point, its center.*

The radius, r, is the distance from the center to a point on the circle. The diameter, d, is the line segment through the center connecting two points on the circle. Here $d = 2r$.

Now, the **area of a circle** is given by $A = \pi r^2$ and the **circumference** is given by $C = \pi d = 2\pi r$.

Example 20. *Find the area and circumference of a circle with $d = 5$ cm.*

$$A = \pi r^2 = \pi \times 5^2 \approx 78.54\ cm^2,$$
$$C - \pi d = \pi \times 5 \approx 15.71\ cm.$$

Example 21. *A rectangular field is to be enclosed by a fence that has a length of 42 yards and a width of 28 yards. If fencing costs \$5.25 per foot, find the cost to enclose the field.*

To find out how much fencing is needed, we need the perimeter so $l = 42$ and $w = 28$. So the perimeter formula can be restated as $P = 2l + 2w$. Thus

$$P = 2(42) + 2(28) = 84 + 56 = 140\ yds.$$

Now to find the cost of the material, one should see that we have the price per foot and the perimeter is in yards, so we need to convert to feet. Recall that 1 yards is 3 feet, so

$$\frac{140\,yds}{1} \cdot \frac{3\,ft}{1\,yd} = 140 \times 3 = 420\,ft.$$

Finally the cost is just $420\,ft \times \$5.20$ *per foot* $= \$2205$.

Example 22. *A rectangular field is four times as long as it is wide. If it has a perimeter of 550 feet. What is the length and width?*

First, we see that we can write the length in terms of the width, four times as long as it is wide, meaning $l = 4w$. *Thus subbing into the formula* $P = 2l + 2w$ *yields*

$$P = 2l + 2w = 2(4w) + 2w = 10w.$$

Subbing in $P = 550$ *yields*

$$w = \frac{550}{10} = 55\,ft.$$

Lastly, to find l, use $l = 4w = 4(55) = 220$ *feet.*

Example 23. *Find the cost of flooring a 12 foot by 15 foot floor with carpet costing* $\$18.50$ *per square yard. First note that our cost is in per square yard and our dimensions are in feet. Therefore we need to convert to yards first by using the conversion 3 feet to 1 yard.*

Using an appropriate dimensional analysis argument, we have that the floor has dimensions 4 yards by 5 yards. Thus since the area of a rectangle is $A = bh$ *where* $b = 4$ *and* $h = 5$, *we have that*

$$A = (4)(5) = 20 \text{ square yards}.$$

Lastly, the cost is just 20 *sq yds* $\times \$18.50$ *per square yds* $= \$370$.

Example 24. *Which is the better buy? A 16 inch diameter pizza for* $\$15$ *or an 8 inch diameter pizza for* $\$7.50$.

To answer this question we need to find the lowest cost per square inch. Meaning we need to find the areas of the pizzas first. For the large we have

$$A_{Lg} = \pi r^2 = \pi(8)^2 = 64\pi \approx 201 \text{ sq in},$$

$$A_{Md} = \pi r^2 = \pi(4)^2 = 16\pi \approx 50 \text{ sq in}.$$

Thus to find the cost per square inch, all we need to do is divide the cost by the area of the pizza (since we want cost per sq inch), so

$$\frac{\$15}{201\ in^2} \approx \$.07 \text{ per } in^2$$

$$\frac{\$7.50}{50\ in^2} \approx \$.15 \text{ per } in^2.$$

Since the large has the smallest cost per square inch, the large is the better buy.