Study Guide for Exam 1 in MATH 2420

Section 1.2

Definitions, Theorems, etc from the section:

1. If S is a set, the notation $x \in S$ means that x is an element of S. The notation $x \notin S$ means that x is not an element of S.

2. A set S may be specified in **set-roster notation** meaning that all the elements in S are written between braces $\{...\}$.

3. **Axiom of Extension:** a set is completely determined by what its elements are, not the order nor whether or not they are repeated more than once.

4. $\mathbb{R}$ represents the set of all real numbers. Note that we say "nonneg" in the superscript to mean we include zero, and + if we want the numbers strictly greater than 0.

5. $\mathbb{Z}$ represents the set of all integers. Note that we say "nonneg" in the superscript to mean we include zero, and + if we want the numbers strictly greater than 0.

6. $\mathbb{Q}$ represents the set of rational numbers. Note that we say "nonneg" in the superscript to mean we include zero, and + if we want the numbers strictly greater than 0.

7. Let S denoted a set and $P(x)$ be some property that the elements of S may or may not apply. The set defined as **the set of all elements $x \in S$ such that $P(x)$ is true** is denoted by

$$\{x \in S | P(x)\}.$$

8. Given two sets A and B, we say that A **is a subset of** B, $A \subseteq B$ if and only if every element of A is also in B, i.e. $A \subseteq B$ means if $x \in A$ then $x \in B$.

9. Given two sets A and B, we say that A **is a proper subset of** B, $A \subset B$ if and only if every element of A is also in B and there exists at least one element of B not in A, i.e. $A \subset B$ means if $x \in A$ then $x \in B$ and there exists an $y \in B$ such that $y \notin A$.

10. Given a positive integer n and let $x_1, \ldots, x_n$ be elements. The **ordered n-tuple** $(x_1, \ldots, x_n)$ consists of $x_1, \ldots, x_n$ together with an ordering, x_1 first, x_2 second, and so on. Remember that $(x_1, x_2, \ldots, x_n) = (y_1, y_2, \ldots, y_n)$, means that $x_1 = y_1$, $x_2 = y_2$, and so on.

11. Let n be a positive integer and A a finite set, then the **string of length n over A** is an order n-tuple of elements of A WITHOUT writing parentheses or commas. The **null string** over A is defined to be the string with no characters i.e. length zero denoted λ. If $A = \{0, 1\}$, the any string over A is called a bit string.

Problems to be able to Solve:

1. Determine if two sets are equal, subsets, or proper subsets.
2. Convert the set-builder notation of a set to English.
3. Write out the elements in a set in roster notation or an equivalent way, i.e. interval notation or number line.
4. Determine if the correct symbol is being used in statements involving $\in$, $\notin$, $\subset$, $\subseteq$.
5. Determine if ordered n-tuples are equivalent.
6. Any questions or problems discussed in lecture/practices/in-class assignment, and WebAssign.

Section 2.1

Definitions, Theorems, etc from this section:

1. An **argument** is a sequence of statements that demonstrate the truth of an assertion (we get a better definition later).
2. A **statement**, also called a proposition, is a sentence that is either true or false, but not both. Most times we use place holders, p, q, r to serve in place of the component statements, these are called statement variables.
3. **Logic** is the science of reasoning.
4. The symbol $\sim$ represents the word NOT, and is called **negation**. Remember that negation negates the thing immediately after the symbol so watch out for how things are structured, i.e. $\sim q \vee p$ is different from $\sim (q \vee p)$.
5. The symbol $\vee$ represents the word OR, so $p \vee q$ means p or q.
6. The symbol $\wedge$ represents the word AND, so $p \wedge q$ means p and q. Remember p but q means p and q.
7. Remember that neither p nor q means $\sim p$ and $\sim q$.
8. The symbol $\therefore$ represents the word THEREFORE.
9. Remember the equivalence of OR and AND in regards to the inequality symbols

$$x \leqslant a \qquad \text{equivalent to } x < a \text{ or } x = a.$$

$$a \leq x \leqslant b \qquad \text{equivalent to } a \leqslant x \text{ and } x \leqslant a.$$

Similar for the symbols in the opposition direction.

10. If p is a statement variable, then the **negation** of p is "not p or "It is not the case that p' and is denoted $\sim p$. Truth table given by:

p	$\sim p$
T	F
F	T

11. If p and q are statement variables, the **conjunction** of p and q is "p and q," denoted $p \wedge q$. Truth table given by:

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Notice that the conjunction is ONLY TRUE when BOTH p and q are true.

12. **Inclusive OR**- means we can have p or q or BOTH.

13. **Exclusive OR**- means we can have p or q BUT NOT BOTH. To specify this OR, we would use the phrase "p or q but not both.

14. If p and q are statement variables, the **disjunction** of p and q is p or q denoted $p \vee q$ and is referring to the INCLUSIVE OR. Truth table given by:

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Notice that the OR is ONLY FALSE when BOTH p and q are false.

15. A **statement form** is an expression that is made up of statement variables and logical connectives that becomes a statement when statements are substituted for the variables. We are interested in the truth tables of statement forms since it lists the truth values for all possible combinations of true/false for the component statements (p, q, r).

16. We say that two statement forms are **logically equivalent** if and only if they have the identical truth values for each possible substitution of statements for their statement variables. This is denoted $P \equiv Q$ for statement forms P and Q. In practice, we look at the columns of P and Q in their truth tables and their columns should be the same for each combination of T/F for the statement variables.

(a) Construct a truth table with one column for the truth values of P and another column for the truth values of Q.

(b) Check each combination of truth values of the statement variables to see whether the truth value of P is equal to the truth value of Q.

i. If in each row the truth value of P equals that of Q, then they are equivalent.

ii. If in at least one row te truth value of P differs from that of Q, then they are NOT equivalent.

17. Double Negation: $\sim(\sim p) \equiv p$.

18. DeMorgan's Law: (1) $\sim(p \wedge q) \equiv \sim p \vee \sim q$ (2) $\sim(p \vee q) \equiv \sim p \wedge \sim q$.

19. A **tautology** is a statement form that is ALWAYS TRUE regardless of the combination of truth values of the individual statements, typically denoted t. Easiest example is $p \vee \sim p$.

20. A **contradiction** is a statement form that is ALWAYS FALSE regardless of the combination of the truth values of the individual statements, typically denoted c. Easiest example is $p \wedge \sim p$.

21. ALL THE LAWS!

(a) Commutative Law: (1) $p \wedge q \equiv q \wedge p$ (2) $p \vee q \equiv q \vee p$.

(b) Associative Law: (1) $p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r$ (2) $p \vee (q \vee r) \equiv (p \vee q) \vee r$.

(c) Distributive Law: (1) $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ (2) $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$.

(d) Identity Law: $p \wedge t \equiv p$ $p \vee c \equiv p$.

(e) Negation Law: $p \vee \sim p \equiv t$ $p \wedge \sim p \equiv c$.

(f) Double Negation: $\sim(\sim p) \equiv p$.

(g) Idempotent Laws: $p \wedge p \equiv p$ $p \vee p \equiv p$.

(h) DeMorgan's Law: (1) $\sim(p \wedge q) \equiv \sim p \vee \sim q$ (2) $\sim(p \vee q) \equiv \sim p \wedge \sim q$.

(i) Absorption Law: $p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$

(j) Negation of T and C: $\sim t \equiv c$ $\sim c \equiv t$.

Problems to be able to Solve:

1. Create truth tables for statement forms.
2. Translate statements between symbolic and English.
3. Determine if statement forms are logically equivalent.
4. Show statements are equivalent to other statements through the use of the laws.
5. Any questions or problems discussed in lecture/practices/in-class assignment, and WebAssign.

Section 2.2

Definitions, Theorems, etc from this section:

1. If p and q are statement variables, the **conditional** of q by p is "If p, then q" or "p implies q, is denoted $p \to q$ Where o is the **hypothesis/antecedent** and q is the **conclusion/consequent**. Truth table is given by:

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

2. Remember that if p then q is equivalent to not p or q.
3. Remember the negation of conditional $\sim (p \to q) \equiv p \wedge \sim q$.
4. There are three different variations on the conditional, $p \to q$.
   (a) Converse: flips the antecedent and consequent. $q \to p$.
   (b) Inverse: negates the consequent and antecedent, but keeps the order. $\sim p \to \sim q$.
   (c) Contrapositive: flips and negates the antecedent and consequent. $\sim q \to \sim p$.
5. Remember that the CONVERSE and INVERSE are logically equivalent.
6. Remember that the CONDITIONAL and CONTRAPOSITIVE are logically equivalent.
7. Remember that the statement p only if q is equivalent to if not q, then not p (contrapositive) which is also equivalent to the conditional $p \to q$.
8. If p and q are statement variables, the **biconditional** of p and q, "p if and only if q, is denoted $p \longleftrightarrow q$. Truth table is given by:

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

9. Remember the biconditional is equivalent to $p \leftrightarrow q \equiv (p \to q) \wedge (q \to p$.
10. If r and s are statements, then r **is a sufficient condition** for s means $r \to s$, i.e. "r occurring is enough for s to occur."

11. If r and s are statements, then r **is a necessary condition** for s means $\sim r \to \sim s$, i.e. meaning that r occurring is required for s to occur.

12. Remember a r is necessary and sufficient for s, is the biconditional.

Problems to be able to Solve:

1. Create truth tables for statement forms.
2. Translate statements between symbolic and English.
3. Determine if statement forms are logically equivalent.
4. Show statements are equivalent to other statements through the use of the laws.
5. Given a conditional, produce the converse, inverse, and contrapositive.
6. Any questions or problems discussed in lecture/practices/in-class assignment, and WebAssign.

Section 2.3

Definitions, Theorems, etc from this section:

1. An **argument** is a sequence of statements (in English), and an argument is a sequence of statement forms (in symbolic). All statements in and argument and all statements in an argument form with the exception of the last one, i.e. before the $\therefore$, are called **premises**. The final statement, i.e. after the $\therefore$ is called the **conclusion**.

2. An argument form is **valid** if no matter what statements are substituted, we have that if the premises are true then the conclusion is true. To determine validity:

   (a) Identify the premises and conclusion.

   (b) Construct a truth table showing all the truth values of the premises and conclusion.

      i. A row in which ALL premises are true is called a **critical row**.

      ii. If there is A critical row where the conclusion is false, then the argument is INVALID.

      iii. If EVERY critical row has a true conclusion then the argument is VALID.

3. An argument with two premises, the first one called the major premises and the second the minor premise, is called a **syllogism**.

4. Modus Ponens, also called affirming the antecedent, is a valid argument form of the form:

   $p \to q$

   p

   $\therefore q$

5. Modus Tollens, also called denying the consequent, is a valid argument form of the form:

   $p \to q$

   $\sim q$

   $\therefore \sim p$

6. A **rule of inference** is a valid argument form, of which the following are valid as well (and Modus Tollens and Modus Ponens).

7. Generalization, we have a specific statement as a premise and the conclusion is more general and uses the OR connective, I will only show one type:

   p

   $\therefore p \vee q$

8. Specialization, we have a broad statement that uses the AND connective as a premise and the conclusion is one part of the conjunction, I will only show one type:

   $p \wedge q$

   $\therefore q$

9. Elimination, we have two possibilities connected by an OR and the minor premise that rules one out, therefore the conclusion is the other one, I will only show one type:

   $p \vee q$

   $\sim p$

   $\therefore q$

10. Transitivity, we have a string of if- then statements where one implies the second, the second implies the third and so on, therefore the first implies the last statement:

    $p \to q$

    $q \to r$

    $\therefore p \to r$

11. Division of Cases, an argument when one premise is an OR statement. If you can show that that in either case a specific conclusion follows, then the conclusion must be true:

$p \vee q$

$p \to r$

$q \to r$

$\therefore r$

12. A **fallacy** is an error in reasoning that results in an invalid statement. This typically stems from ambiguous premises that are treated as unambiguous, circular reasoning, or jumping to conclusions.

13. Converse Error, also called affirming the consequent, is a FALLACY of the form:

    $p \to q$

    q

    $\therefore p$

14. Inverse Error, also called denying the antecedent, is a FALLACY of the form:

    $p \to q$

    $\sim p$

    $\therefore \sim q$

15. Converse Error, also called affirming the consequent, is a FALLACY of the form:

    $p \to q$

    q

    $\therefore p$

16. In some case we are able to show that the statement "not p" leads to a contradiction, therefore p must be true. This is the contradtion rule.

    $\sim p \to c$

    $\therefore p$

Problems to be able to Solve:

1. Identifying different argument forms as rules of inference or fallacies.
2. Show whether an argument is valid or invalid.
3. Deduce a conclusion or verify a conclusion if given the premises by using the rules of inference.
4. Translate an argument to its argument form.
5. And questions of problems discussed in lecture/practices/in-class assignment, and WebAssign.

Section 2.5

Definitions, Theorems, etc from this section:

1. A number written in base 10, is written in **decimal notation**, where

$$X_{10} = d_n \times 10^n + d_{n-1} \times 10^{n-1} + \cdots + d_1 \times 10^1 + d_0 \times 10^0$$

   where $d \in \{0, 1, 2, ..., 9\}$

2. A number written in base 2, is written in **binary notation**, where

$$X_2 = d_n \times 2^n + d_{n-1} \times 2^{n-1} + \cdots + d_1 \times 2^1 + d_0 \times 2^0$$

   where $d \in \{0, 1\}$

3. A number written in base 16, is written in **hexadecimal notation**, where

$$X_{16} = d_n \times 16^n + d_{n-1} \times 16^{n-1} + \cdots + d_1 \times 16^1 + d_0 \times 16^0$$

   where $d \in \{0, 1, 2, ..., 15\}$. Remember 10-15 are actually A-F.

4. The **8-bit two's complement** of an integer a between -128 and 127 is the 8-bit binary representation given by:

   (a) a, if $a \geq 0$

   (b) $2^8 - |a|$, if $a < 0$

   Remember that the leftmost bit is the sign in this case and 8-bits means we have a string of 8 numbers (0 or 1). Now, the easier way was as follows:

   (a) Write the 8-bit binary representation of $|a|$, don't forget to add leading zeroes if need be.

   (b) Switch all 0's to 1's and 1's to 0's.

   (c) Add 1_2 to the new number.

Decimal	Hexadecimal	4-bit Binary Equivalent
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
10	A	1010
11	B	1011
12	C	1100
13	D	1101
14	E	1110
15	F	1111

Problems to be able to Solve:

1. Convert between the different bases.
2. Add and subtract binary numbers.
3. Find the 8-bit two's complement.
4. Any questions or problems discussed in lecture/practices/in-class assignment, and WebAssign.